8.2: Trigonometric Integrals

Strategies for Evaluating $\int \sin^m x \cos^n x \, dx$

If one or both of the powers are odd:

Use the identity $\cos^2 x = 1 - \sin^2 x$ or $\sin^2 x = 1 - \cos^2 x$

to rewrite the integral as $\int \sin^m x \left(1 - \sin^2 x\right)^k \cos x \, dx$ or $\int \left(1 - \cos^2 x\right)^k \cos^n x \sin x \, dx$.

Then integrate by substitution.

If both powers are even:

Use the half-angle identities.

$$\sin^2 x = \tfrac{1}{2}\left(1 - \cos 2x\right) \text{ and } \cos^2 x = \tfrac{1}{2}\left(1 + \cos 2x\right)$$

Sometimes it is useful to use the identity

$$\sin x \cos x = \tfrac{1}{2} \sin 2x$$

Example 1: Find $\int \sin^2 x \cos^3 x \, dx$.

Example 2: Find $\int \sin^5 x\,dx$.

Example 3: Find $\int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$

Example 4: Find $\int \sin^2 x \cos^4 dx$

Strategies for Evaluating $\int \tan^m x \sec^n x\, dx$

1) Power of secant is even $(n = 2k)$:

Save one factor of $\sec^2 x$ and use the identity $\sec^2 x = 1 + \tan^2 x$ to rewrite the remaining factors in terms of $\tan x$.

$$\int \tan^m x \sec^{2k} x\, dx = \int \tan^m x \left(\sec^2 x\right)^{k-1} \sec^2 x\, dx$$
$$= \int \tan^m x \left(1 + \tan^2 x\right)^{k-1} \sec^2 x\, dx$$

Then substitute $u = \tan x$.

2) Power of tangent is odd $(m = 2k+1)$:

Save one factor of $\sec x \tan x$ and use the identity $\tan^2 x = \sec^2 x - 1$ to rewrite the remaining factors in terms of $\sec x$.

$$\int \tan^{2k+1} x \sec^n x\, dx = \int \left(\tan^2 x\right)^k \sec^{n-1} x \sec x \tan x\, dx$$
$$= \int \left(\sec^2 x - 1\right)^k \sec^{n-1} x \sec x \tan x\, dx$$

Then substitute $u = \sec x$.

Example 5: Find $\int \tan^4 x \sec^4 x \, dx$.

Example 6: Find $\int \tan^3 x \sec^3 x \, dx$.

Example 7: Find $\int_0^{\pi} \sqrt{1-\cos 6x}\, dx$.

Strategies for Evaluating $\int \sin mx \cos nx\, dx$, $\int \sin mx \sin nx\, dx$, and $\int \cos mx \cos nx\, dx$

Use one of the following identities.

$$\sin A \cos B = \tfrac{1}{2}\left[\sin(A-B)+\sin(A+B)\right]$$
$$\sin A \sin B = \tfrac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]$$
$$\cos A \cos B = \tfrac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$$

Example 8: Find $\int \sin 6x \sin 3x\, dx$.

Example 9: $\int \csc x \, dx$

Example 10: $\int \frac{1}{1-\cos x} dx$