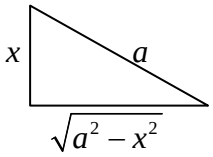
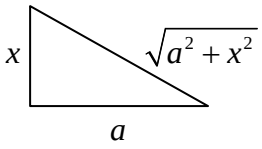
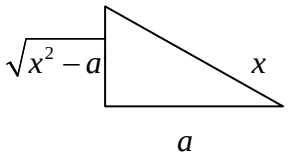
8.3: Trigonometric Substitution

Trigonometric substitution can be useful in evaluating integrals of certain radical expressions.

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$, with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ 	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$, with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ 	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$, with $0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$ 	$\sec^2\theta - 1 = \tan^2\theta$

Example 1: $\int \frac{x}{\sqrt{100 - x^2}}\,dx$

Example 2: $\int \frac{dx}{x^2\sqrt{x^2-4}}$

Example 3: $\int \sqrt{9+x^2}\,dx$

Example 4: $\int x^3\sqrt{x^2+4}\,dx$

Example 5: $\int \frac{dx}{\sqrt{x^2-6x+5}}$