# 7.2: Sets

Definition: A *set* is a well-defined collection of objects. Each object in a set is called an *element* of that set.

Examples of sets:

Not sets:

Sets can be finite or infinite.

Examples of finite sets:

Examples of infinite sets:

Notation:

- We usually use capital letters for sets.
  We usually use lower-case letters for elements of a set.
- $a \in A$ means *a* is an element of the set *A*.
  $a \notin A$ means *a* is not an element of the set *A*.
- The *empty set* is the set with no elements. It is denoted $\varnothing$. This is sometimes called the *null set*.
- $S = \{x \mid P(x)\}$ means "*S* is the set of all *x* such that $P(x)$ is true". (called rule notation or set roster notation).

Example: $S = \{x \mid x$ is an even positive integer$\}$ means $S = \{2, 4, 6, 8, \ldots\}$

Definition: We say two sets are *equal* if they have exactly the same elements.

**Subsets:**

Definition: If each element of a set *A* is also an element of set *B*, we say that *A* is a *subset* of *B*. This is denoted $A \subseteq B$ or $A \subset B$. If *A* is not a subset of *B*, we write $A \not\subset B$.

Definition: We say *A* is a *proper subset* of *B* if $A \subseteq B$ but $A \neq B$. (In other words, every element of *A* is also an element of *B*, but *B* contains at least one element that is not in *A*.)

Note on notation: Some books use the symbol $\subset$ to indicate a proper subset. Some books use $\subset$ to indicate any subset, proper or not.

Definition: The set of all elements under consideration is called the *universal set*, usually denoted *U*.
Example: If you're dealing with sets of real numbers, then *U* is the set of all real numbers. So "Wednesday" would not be an element of *U*, but 5.7 would be in *U*.

**Example 1:** Consider these sets.

$$A = \{1,2,3,4,5,6\}$$
$$B = \{1,2,3,4,5,6,7,8\}$$
$$C = \{1,3,5,2,4,6\}$$

Note:
- $\varnothing$ is a subset of every set. (i.e. $\varnothing \subseteq A$ for every set *A*.)
- Every set is a subset of itself. (i.e. $A \subseteq A$ for every set *A*.)

**Example 2:** List all subsets of $\{1,2,3\}$.

Note: If a set has *n* elements, how many subsets does it have?

**Set operations:**

- Union $\cup$ : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Intersection $\cap$ : $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Complement $A'$ or $A^C$ or $A^{\sim}$: $A' = \{x \in U \mid x \notin A\}$.

Note: $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.
$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.

Definition: We say that $A$ and $B$ are *disjoint sets* if $A \cap B = \varnothing$.

**Example 3:** $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$H = \{1, 3, 5, 7\}$

$K = \{1, 2, 3\}$

$J = \{2, 4, 6, 8\}$

$L = \{1, 2\}$

**Venn Diagrams:** These help us visualize set relationships and operations.

**Example 4:** Draw Venn diagrams for $A \cup B$, $A \cap B$, $A'$, $(A \cap B)'$, $A' \cap B'$, and $A' \cap B$.

**Example 5:** On a Venn diagram, shade $A \cup B \cup C$, $A \cap B \cap C$, and $(A \cup B') \cap C$.

**Example 6:** Consider a group of students. 30 of them are enrolled in a math course and 35 are enrolled in an English course. 13 of the students are enrolled in an English course and also a math course. How many students are enrolled in math or English?

<u>Notation</u>: $n(A)$ means the number of elements in set *A*.

<u>Addition principle for Counting</u>

For any two sets *A* and *B*,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

If *A* and *B* are disjoint ($A \cap B = \varnothing$), then $n(A \cup B) = n(A) + n(B)$.

**Example 7:** 100 students are surveyed to determine if they had watched ESPN or Fox Sports Channel in the last 3 months. The results show that 65 students watched ESPN, 55 watched Fox Sports, and 30 watched neither.

a. How many people watched ESPN but not Fox Sports?
b. How many people watched Fox Sports but not ESPN?
c. How many watched both networks?

**Example 8:** I want to buy a car from Jay Austin Motors. Of all the cars on the lot, 89 cars have power windows, 100 have moonroofs, and 74 have CD changers. 32 cars have both power windows and CD changers, 40 have both a moon roof and a CD changer, and 53 have a moonroof and power windows. 12 cars have all three features, and 21 cars are base models with none of the features.

I passionately dislike power windows, but I would like a moonroof and a CD changer. How many cars do I have to choose from?

How many cars are on Jay Austin's lot?