8.5: Random Variable, Probability Distribution, and Expected Value

A *random variable* is a function that assigns a real number to each outcome of an experiment.

A *probability distribution* is a function that assigns a probability to each outcome. If there are a finite number of outcomes, the sum of all their probabilities must equal 1. Each probability must be between 0 and 1, inclusive.

This means that the probability distribution assigns a probability to each value of the random variable.

Expected value**:**

Suppose that a random variable *x* can take on the *n* values $x_1, x_2, ..., x_n$. Suppose the associated probabilities are $p_1, p_2, ..., p_n$. Then the expected value of *x* is

$$E(x) = x_1 p_1 + x_2 p_2 + ... + x_n p_n.$$

Example 1: A probability distribution is given by the table below. Find the expected value.

x	3	4	5	6	7	8	9
$P(x)$	0.15	0.20	0.30	0.12	0.08	0.10	0.05

Example 2: Write the probability distribution associated with the rolling of two dice, where x represents the sum of the numbers on the dice. Draw a histogram to represent the distribution. What is the expected value of the sum when two dice are rolled?

<u>Example 3:</u> Suppose that an organization sells 1000 raffle tickets for $1 each. One ticket is for an IPod worth $200, and three tickets are for $50 gift certificates to a restaurant. Find the expected net winnings for a person who buys one ticket.

<u>Example 4:</u> Suppose the yearly premium for a car insurance policy is $2300 for a customer in a certain category. Statisticians for the insurance company have determined that a person in this category has a 0.007 probability of having an accident that costs the insurance company $100,000 and a 0.015 probability of having an accident that costs the insurance company $30,000. What is the expected value of the insurance policy to the customer? To the insurance company?