

3.2: Compound Interest

If at the end of a payment period, the interest due is reinvested at the same rate, then the interest as well as the principal will earn interest. This is called *compound interest*. The interest is paid into the account at the end of each compounding period.

Example 1: Suppose you invest \$1000 compounded quarterly at an annual interest rate of 8%. How much money will you have after one year?

Compound Interest:

$$\begin{aligned} A &= P(1+i)^n \\ &= P\left(1+\frac{r}{m}\right)^n \end{aligned}$$

where

$i = \frac{r}{m}$ is the interest rate per compounding period

r = annual interest rate

m = number of compounding periods per year

n = total number of compounding periods

P = principal (present value)

A = amount (future value) at the end of n compounding periods.

Example 2: What is the future value of \$1000 after 8 years at 6% compounded monthly?

Example 3: How much should I invest now at 4% interest compounded monthly in order to have \$10,000 in 6 years?

Example 4: You decide to invest some money so that you will have \$1,000,000 on your 75th birthday. At 8% compounded quarterly, how much should you invest on your 25th birthday?

Example 5: How long will it take \$5,000 to grow to \$7,000 if it is invested at 8% compounded monthly?

Example 6: How long will it take money to double if it is invested at 7.5% compounded monthly?

Effective rates:

The effective rate, sometimes called the *annual percentage yield*, converts a compound interest rate to an equivalent simple interest rate. This allows us to compare interest rates which have different compounding periods.

Effective Rate:

The effective rate, or annual percentage yield (APY), is given by

$$r_e = APY = \left(1 + \frac{r}{m}\right)^m - 1,$$

where

r = annual interest rate

m = number of compounding periods per year

Example 7: What is the effective rate for money invested at 6% compounded quarterly?

Example 8: Which investment is better, Note A at 9% compounded monthly or Note B at 9.2% compounded semiannually?