4.1: Systems of Linear Equations

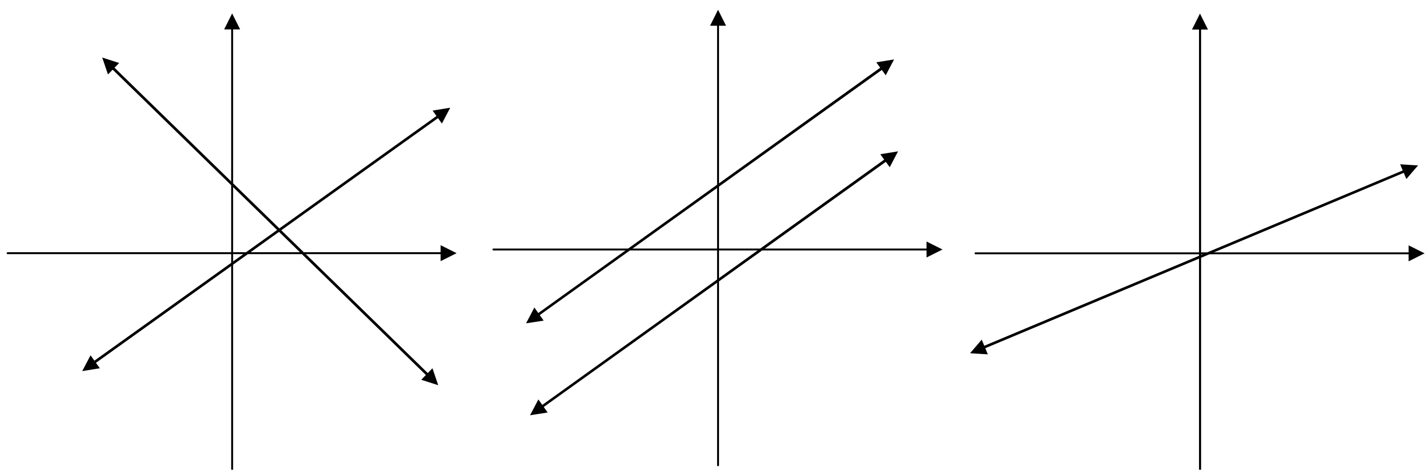
System of equations: a set of equations with common variables.

A pair of lines can be given as a system of equations of the form $\begin{cases} ax+by=c \\ dx+ey=f \end{cases}$.

The graph of each equation in the system is a straight line.

Three possibilities for the number of solutions:

One Solution (lines intersect at a single point) Consistent, independent	No Solution (lines are parallel) Inconsistent	Infinitely Many Solutions (lines coincide) Consistent, dependent



In a nutshell:

- "Consistent" means the system can be solved.
- "Inconsistent means it cannot be solved.
- "Independent" means the equations are not multiples of one another.
- "Dependent" means the equations are multiples of one another. They're the same line, just written in a different format.

Methods for solving a system of linear equations:

- Substitution
- Elimination
- Graphing (doesn't give exact solutions)
- Matrices

<u>Example 1:</u> A movie theater sells regular and student tickets. The cost of two regular tickets and one student ticket is $16. The cost of three regular tickets and three student tickets is $30. What is the price of each type of ticket?

Set up the system:

Solve by substitution:

<u>Step 1</u>: Solve one of the equations for one of the variables.
<u>Step 2</u>: "Substitute" the result from Step 1 into the *other* equation.
<u>Notice</u>: You end up with one equation in one variable only.
<u>Step 3</u>: Solve this resulting equation for the other variable.
<u>Step 4</u>: Substitute the result from Step 3 into one of the original equations (it doesn't matter which one). Solve this equation for the remaining variable.
<u>Step 5</u>: Check!

Solve by elimination:

Our goal is to eliminate one of the variables by adding the equations (or multiples of the equations) together.

Step 1: Choose a variable to eliminate. Multiply each equation by an appropriate number. Choose these numbers so that when you add the resulting equations, your chosen variable will go away.
Step 2: Add the resulting equations. This will give you one equation in one variable.
Step 3: Solve this new equation for its variable.
Step 4: Substitute the result from Step 3 into one of the original equations (it doesn't matter which one). Solve this equation for the remaining variable.
Step 5: Check!

The above example had a unique solution. What about cases where there is no solution or an infinite number of solutions?

<u>If there's no solution (inconsistent):</u> You'll get a contradiction (for example, $0 = 17$).

<u>Example 2:</u> Solve the system $\begin{cases} 3y - 2x = 15 \\ 4x - 6y = -6 \end{cases}$.

<u>If there are infinitely many solutions (consistent, dependent) (same line)</u>: You'll get a statement that's always true (usually $0 = 0$). In this case your solution will have one variable written in terms of the other.

<u>Example 3:</u> Solve the system $\begin{cases} 2x + 4y = 6 \\ 3x + 6y = 9 \end{cases}$.

Example 4: A pet supply company makes two types of fancy dog beds: the *Couch* and the *Throne*. Each *Couch* requires 30 minutes in the cutting department and 45 minutes in the sewing department. Each *Throne* requires 45 minutes in cutting and 75 in sewing. If the company has allocated a total of 80 hours of cutting time and 130 hours of sewing time to dog beds, how many of each type can be made?

Example 5: An apparel shop sells shirts for $40 and ties for $15. Its entire stock is worth $14,165, but sales are slow and only half the shirts and one-third of the ties are sold, for a total of $6295. How many shirts and ties are left in the store?