

1.2: Finding Limits Graphically and Numerically

Limit of a function:

Definition of a Limit:

$$\lim_{x \rightarrow a} f(x) = L$$

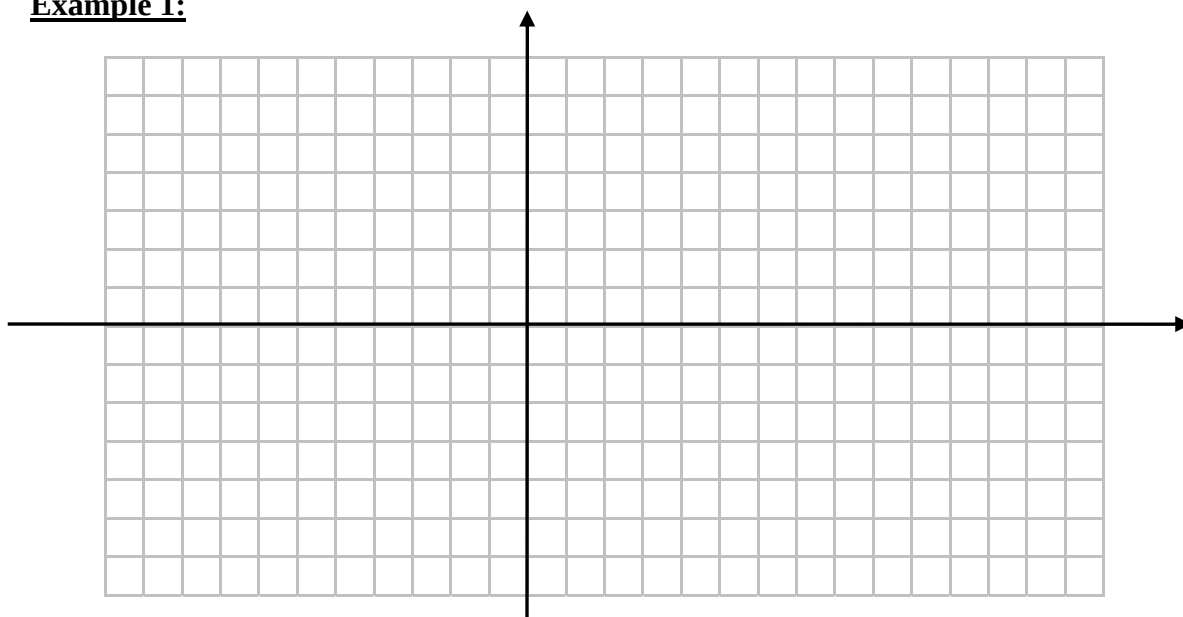
The statement above means that we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a but not equal to a .

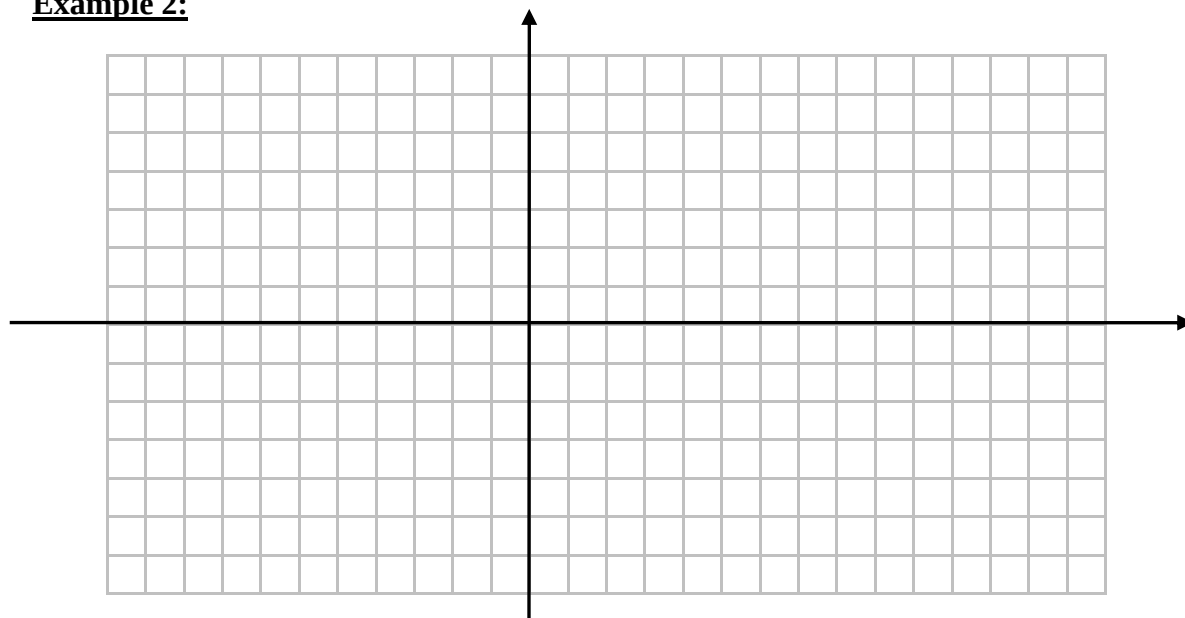
We read this as “the limit of $f(x)$, as x approaches a , is equal to L .”

Alternative notation: $f(x) \rightarrow L$ as $x \rightarrow a$. ($f(x)$ approaches L as x approaches a)

Finding limits from a graph:

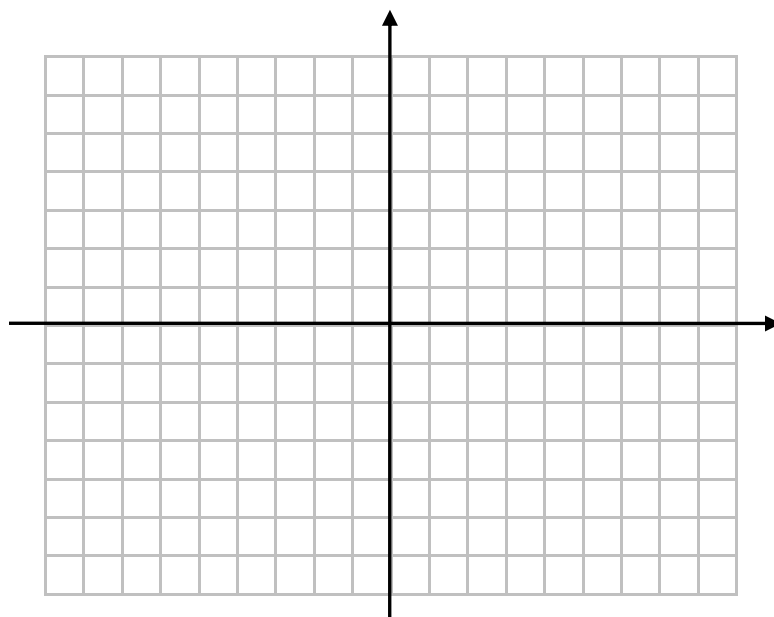
Example 1:



Example 2:

Example 3: Graph the function. Use the graph to determine $\lim_{x \rightarrow 2} f(x)$ and $\lim_{x \rightarrow -1} f(x)$.

$$f(x) = \begin{cases} x-1 & \text{if } x \geq 2 \\ x^2 & \text{if } x < 2 \end{cases}$$



Finding limits numerically:

Example 4: For the function $f(x) = \frac{x-5}{x^2-25}$, make a table of function values corresponding to values of x near 5.

Use the table to estimate the value of $\lim_{x \rightarrow 5} \frac{x-5}{x^2-25}$.

Example 5: Make a table of values and use it to estimate $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$.

Common reasons $\lim_{x \rightarrow c} f(x)$ may not exist:

1. $f(x)$ approaches a different value when approached from the left of c , compared to when approached from the right of c .
2. $f(x)$ increases or decreases without bound as x approaches c .
3. $f(x)$ oscillates between two values as x approaches c .

The formal (epsilon-delta) definition of a limit:

Definition:

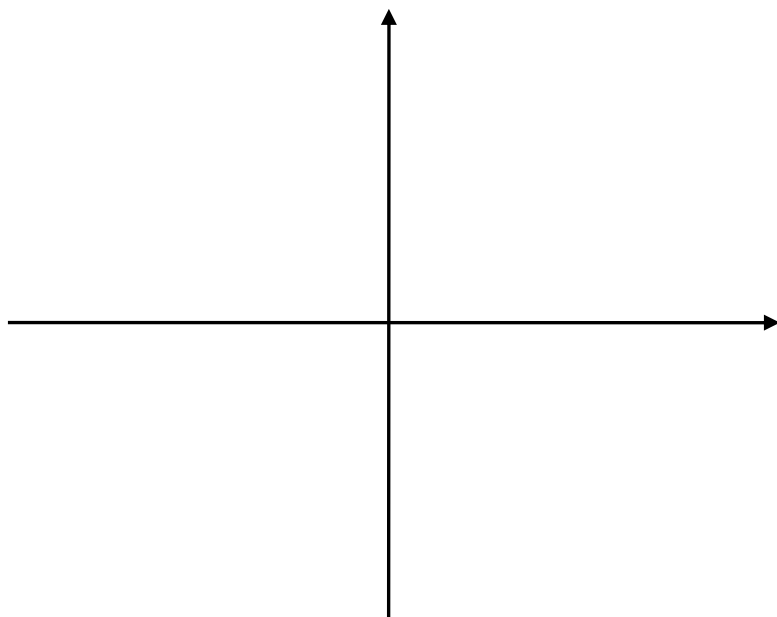
Let f be a function defined on some open interval that contains the number a , except possibly at a itself. Then

$$\lim_{x \rightarrow a} f(x) = L$$

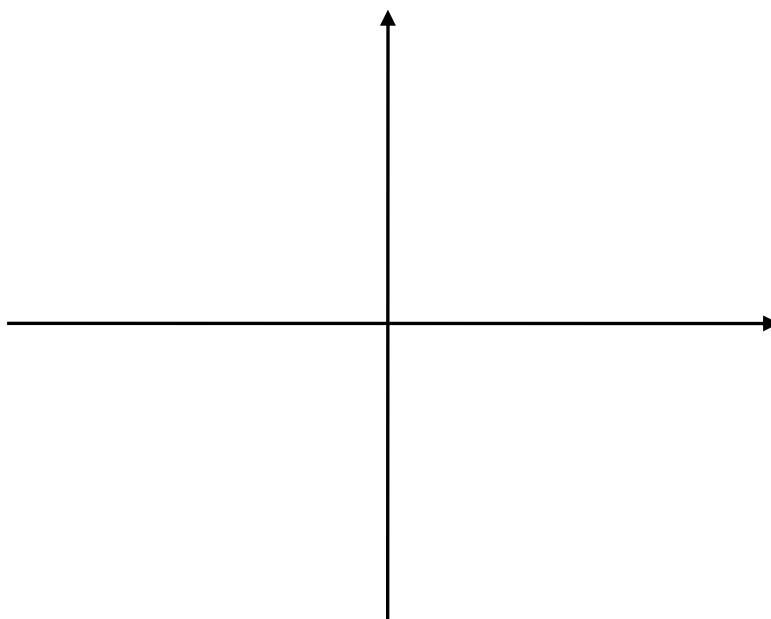
if for every number $\varepsilon > 0$, there is a number $\delta > 0$ such that

$$|f(x) - L| < \varepsilon \text{ whenever } 0 < |x - a| < \delta.$$

Example 6:



Example 7:



Example 8: How close to 3 must we take x so that $6x - 7$ is within 0.1 of 11?

Example 9: How close to 4 must we take x so that $x^2 - 2$ is within 0.01 of 14?

Example 10: Prove that $\lim_{x \rightarrow 4} (2x - 5) = 3$ using the definition of a limit.

Example 11: Prove that $\lim_{x \rightarrow -3} (5x + 1) = -14$ using the definition of a limit.

Example 12: Prove that $\lim_{x \rightarrow 3} x^2 = 9$ using the definition of a limit.

Example 13: Prove that $\lim_{x \rightarrow 2} (x^2 - x + 6) = 8$ using the definition of a limit.