

1.5: Infinite Limits

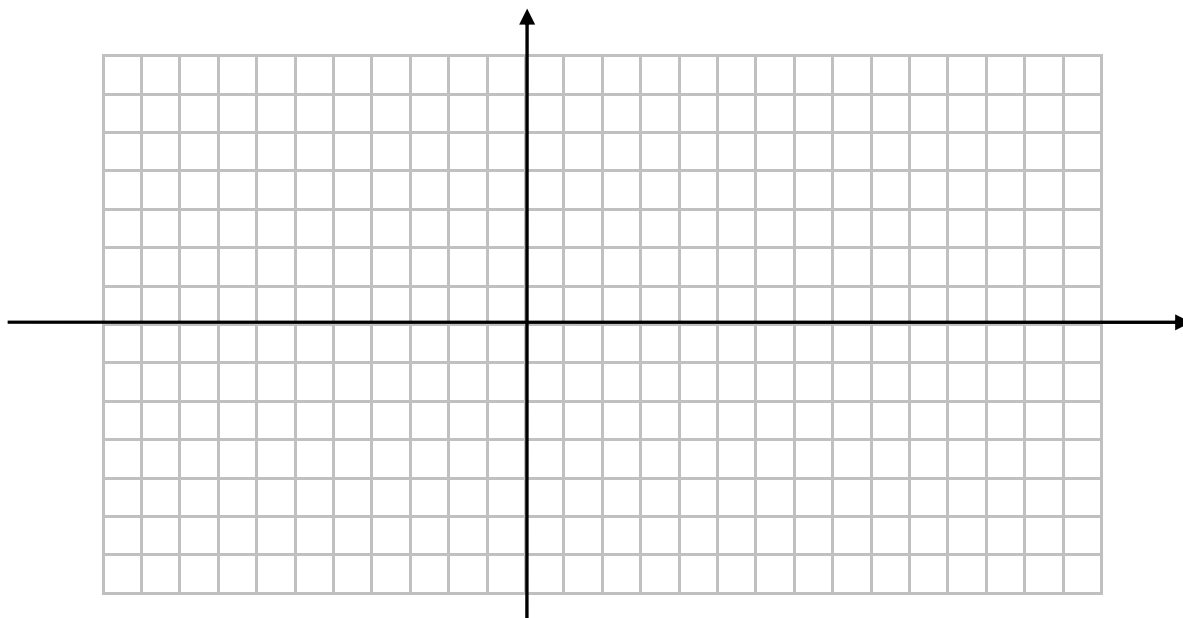
There are two types of limits involving infinity.

Limits at infinity, written in the form $\lim_{x \rightarrow \infty} f(x)$ or $\lim_{x \rightarrow -\infty} f(x)$, are related to horizontal asymptotes and will be covered in Section 3.5, as we learn to graph functions.

Infinite limits take the form of statements like $\lim_{x \rightarrow a} f(x) = \infty$ or $\lim_{x \rightarrow a} f(x) = -\infty$. Infinite limits can result in vertical asymptotes, also important in graphing functions.

Determining infinite limits from a graph:

Example 1:



Determining infinite limits from a table of values:

Example 2: Use a table of values to determine $\lim_{x \rightarrow 2} f(x) = \frac{x+6}{x-2}$.

Example 3: Use a table of values to determine $\lim_{x \rightarrow 2} f(x) = \frac{x+6}{(x-2)^2}$.

Important:

Statements such as $\lim_{x \rightarrow a} f(x) = \infty$, $\lim_{x \rightarrow a} f(x) = -\infty$, or $\lim_{x \rightarrow a^+} f(x) = -\infty$ do NOT mean the limit exists. Rather, these statements mean that the limit DOES NOT EXIST. and they describe the reason that the limit fails to exist (by describing the behavior of the function near the given x -value).

Formal definition of an infinite limit:

Let f be a function defined on some open interval that contains the number a , except possibly at a itself. Then

$$\lim_{x \rightarrow a} f(x) = \infty$$

means that for every positive real number M , there exists a number $\delta > 0$ such that

$$f(x) > M \text{ whenever } 0 < |x - a| < \delta.$$

Similarly, $\lim_{x \rightarrow a} f(x) = -\infty$ means that for every negative real number N , there exists a number $\delta > 0$ such that

$$f(x) < N \text{ whenever } 0 < |x - a| < \delta.$$

Example 4: Prove that $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$.

Evaluating infinite limits from an equation:

Example 5: Determine $\lim_{x \rightarrow 4} \frac{x-8}{x-4}$.

Example 6: Determine $\lim_{x \rightarrow 2^-} \frac{x-8}{x^2-4}$.

Example 7: Determine $\lim_{x \rightarrow 3} \frac{x^3-2}{(x-3)^2}$.

Example 8: Determine $\lim_{x \rightarrow \pi^+} \frac{\sin\left(\frac{x}{3}\right)}{1 + \cos x}$.

Example 9: Determine $\lim_{x \rightarrow \pi/6} \tan(3x)$.

Vertical asymptotes:Vertical Asymptotes:

The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \rightarrow a} f(x) = \infty$$

$$\lim_{x \rightarrow a^+} f(x) = \infty$$

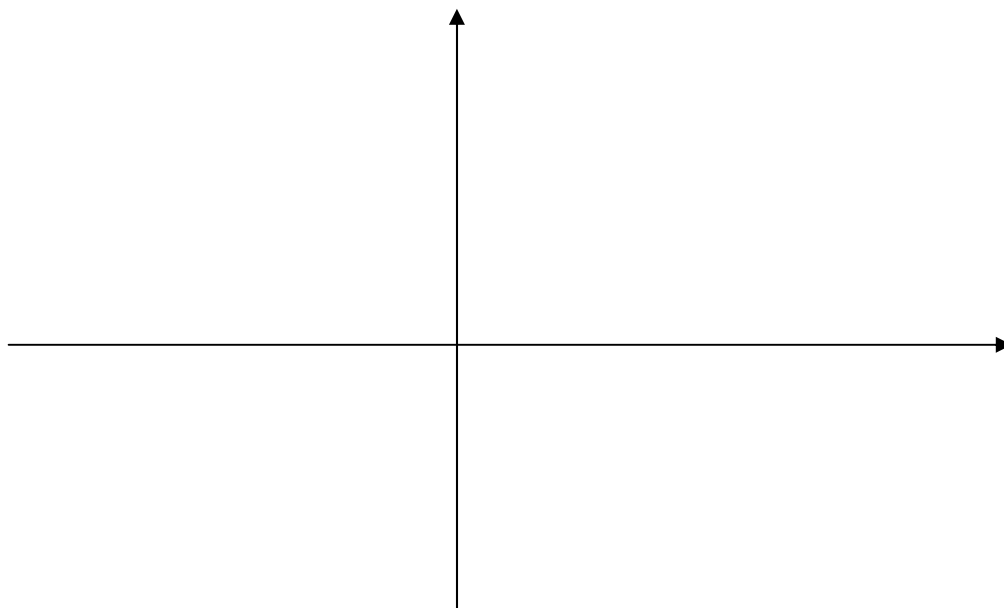
$$\lim_{x \rightarrow a^-} f(x) = \infty$$

$$\lim_{x \rightarrow a} f(x) = -\infty$$

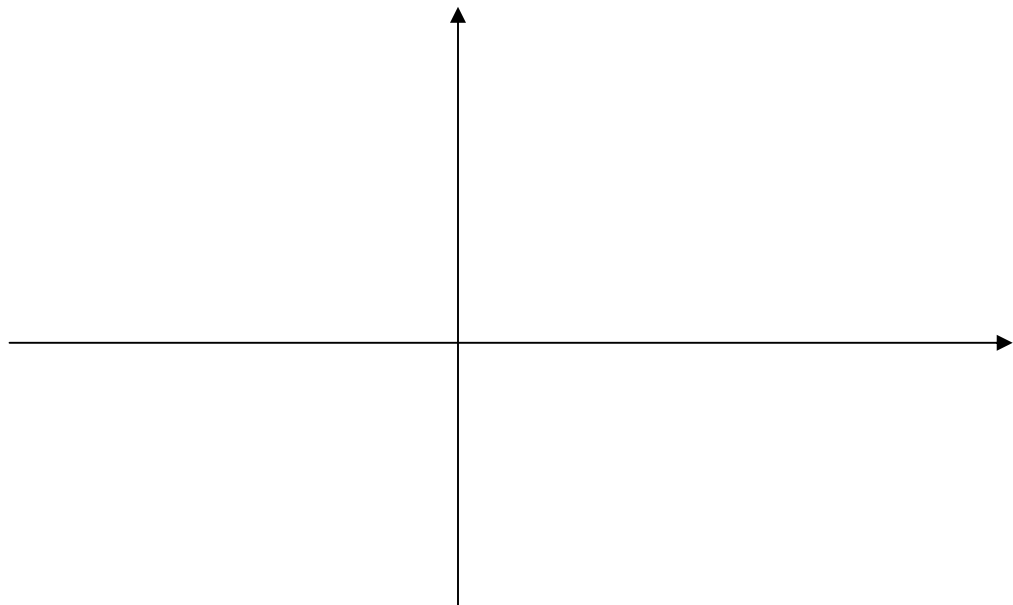
$$\lim_{x \rightarrow a^+} f(x) = -\infty$$

$$\lim_{x \rightarrow a^-} f(x) = -\infty$$

Example 10: Determine the asymptotes of $f(x) = \frac{x-2}{x+3}$. Sketch the graph.



Example 11: Determine the vertical asymptotes of $f(x) = \frac{3}{x^2 - 1}$. Sketch the graph.



Example 12: Determine the vertical asymptotes of $f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$. Sketch the graph.

