# 2.2: Basic Differentiation Rules and Rates of Change

**Basic differentiation formulas:**

1. $\frac{d}{dx}(c) = 0$ for any constant $c$.

2. $\frac{d}{dx}(x^n) = nx^{n-1}$ for any real number $n$.

3. $\frac{d}{dx}\left[cf(x)\right] = c\frac{d}{dx}\left[f(x)\right]$

4. $\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}\left[f(x)\right]+\frac{d}{dx}\left[g(x)\right]$

5. $\frac{d}{dx}\left[f(x)-g(x)\right] = \frac{d}{dx}\left[f(x)\right]-\frac{d}{dx}\left[g(x)\right]$

**Example 1:** Find the derivative of $f(x) = 7$.

**Example 2:** Find the derivative of $f(x) = 5x^3 - x^7 + 12x$.

**Example 3:** Find the derivative of $g(x) = x^{17} + x^{3/2}$.

<u>Recall</u>:

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

$$\frac{1}{x^n} = x^{-n}$$

**<u>Example 4:</u>** Find the derivative of $f(x) = \sqrt[5]{x} + \dfrac{1}{x^2}$.

**<u>Example 5:</u>** Find the derivative of $f(x) = \dfrac{2}{\sqrt[4]{x}}$.

**<u>Example 6:</u>** Find the derivative of $h(x) = \left(\sqrt{x}\right)^5$.

**<u>Example 7:</u>** Find the derivative of $f(x) = -\sqrt[3]{6x^4}$.

**<u>Example 8:</u>** Find the derivative of $f(x) = \dfrac{10}{x^4}$.

**Example 9:** Find the derivative of $g(x)=\dfrac{2\sqrt{x}}{7}$.

**Example 10:** Find the derivative of $f(t)=\dfrac{3}{4t^2}-\sqrt[3]{7t}$ .

**Example 11:** Find the derivative of $f(u)=\dfrac{7u^5+u^2-9\sqrt{u}}{u^2}$.

**Example 12:** Find the equation of the tangent line to the graph of $f(x)=3x-x^2$ at the point $(-2,-10)$.

**Example 13:** Find the point(s) on the graph of $f(x) = x^2 + 6x$ where the tangent line is horizontal.

Definition: The *normal line* to a curve at the point *P* is defined to be the line passing through *P* that is perpendicular to the tangent line at that point.

**Example 14:** Determine the equation of the normal line to the curve $y = \dfrac{1}{x}$ at the point $\left(3, \dfrac{1}{3}\right)$.

**Derivatives of trigonometric functions**:

| | |
|---|---|
| $\dfrac{d}{dx}(\sin x) = \cos x$ | $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$ |
| $\dfrac{d}{dx}(\cos x) = -\sin x$ | $\dfrac{d}{dx}(\sec x) = \sec x \tan x$ |
| $\dfrac{d}{dx}(\tan x) = \sec^2 x$ | $\dfrac{d}{dx}(\cot x) = -\csc^2 x$ |

**Example 15:** Find the derivative of $y = 2\cos x - 4\tan x$.

**Example 16:** Find the derivative of $y = \frac{\sin x}{4} + 3x^4 + \pi^2$.

**Example 17:** Determine the equation of the tangent line to the graph of $y = \sec x$ at the point where $x = \frac{\pi}{4}$.

**Example 18:** Find the points on the curve $y = \tan x - 2x$ where the tangent line is horizontal.

**The derivative as a rate of change:**

The average rate of change of $y = f(x)$ with respect to $x$ over the interval $[x_0, x_1]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_0 + h) - f(x_0)}{h}, \text{ where } h = x_1 - x_0 \neq 0.$$

This is the same as the slope of the secant line joining points $P(x_0, f(x_0))$ and $Q(x_1, f(x_1))$.

The instantaneous rate of change (or, equivalently, just the rate of change) of $f$ when $x = a$ is the slope of the tangent line to graph of $f$ at the point $(a, f(a))$.

Therefore, the instantaneous rate of change is given by the derivative $f\,'$.

**Example 19:** Find the average rate of change in volume of a sphere with respect to its radius $r$ as $r$ changes from 3 to 4. Find the instantaneous rate of change when the radius is 3.

**Example 20:** Find the rate of change of the area of a circle with respect to (a) the diameter; (b) the circumference.

**Velocity:**

If the independent variable represents *time*, then the derivative can be used to analyze motion.

If the function $s(t)$ represents the position of an object, then the derivative $s'(t) = \frac{ds}{dt}$ is the velocity of the object.

(The velocity is the instantaneous rate of change in distance. The average velocity is the average rate of change in distance.)

**Example 21:** A person stands on a bridge 40 feet above a river. He throws a ball vertically upward with an initial velocity of 50 ft/sec. Its height (in feet) above the river after $t$ seconds is $s = -16t^2 + 50t + 40$.

a) What is the velocity after 3 seconds?
b) How high will it go?
c) How long will it take to reach a velocity of 20 ft/sec?
d) When will it hit the water? How fast will it be going when it gets there?

**Example 22:** Suppose a bullet is shot straight up at an initial velocity of 73 feet per second. If air resistance is neglected, its height from the ground (in feet) after t seconds is given by $h(t) = -16.1t^2 + 73t$.

a. The velocity after 2 seconds.
b. How high will the bullet go?
c. When will the bullet reach the ground?
d. How fast will it be traveling when it hits the ground?

**Example 23:** Suppose the position of a particle is given by $f(t) = t^4 - 32t + 7$. What is the velocity after 3 seconds? When is the particle at rest?