

2.4: The Chain Rule

The chain rule allows us to find the derivative of many more functions.

Example 1: Suppose $g(x) = (x^3 + 2)^2$. Find $g'(x)$.

The chain rule lets us differentiate composite functions.

The Chain Rule:

If f and g are both differentiable and $F(x) = f(g(x))$, then F is differentiable and

$$F'(x) = f'(g(x))g'(x).$$

Alternatively, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}.$$

With the chain rule, we take the derivative of the “outer function” and multiply by the derivative of the “inner function”.

Example 2: Find the derivative of $h(x) = (x^3 + 2)^{50}$.

Example 3: Find the derivative of $f(x) = \sqrt{2x^3 - 5x}$.

Example 4: Suppose that $y = \frac{2}{(3x^5 - 4)^3}$. Find $\frac{dy}{dx}$.

Example 5: Find the derivative of $f(x) = \sqrt[3]{\cos x}$.

Example 6: Find the derivative of $g(x) = \cos \sqrt[3]{x}$.

Example 7: Suppose that $y = \frac{1}{\cos x}$. Find $\frac{dy}{dx}$.

Example 8: Suppose that $h(x) = (3x^2 - 4)^3(2x - 9)^2$. Find $h'(x)$.

Example 9: Suppose that $f(x) = \left(\frac{2x+1}{2x-1}\right)^5$. Find $f'(x)$.

Example 10: Find the derivative of $y = \cos(\sin(\pi x^2))$.

Example 11: Find the derivative of $g(x) = \frac{\cos^2 x}{\sqrt{4x+1}}$.

Example 12: Find the first and second derivatives of $f(x) = (x^2 + 4)^5$.

Example 13: Find the first and second derivatives of $f(x) = \cos(3x^2)$.

Example 14: Find the first and second derivatives of $y = \frac{2}{(3x+5)^2}$

Example 15: Find the equation of the tangent line to $y = \frac{x^2}{\sin \pi x + 1}$ at the point where $x = 1$.