

2.5: Implicit Differentiation

Example 1: Given the equation $x^3 - 4y - 9x^2 = 5$, find $\frac{dy}{dx}$ by

- a) Solving explicitly for y .
- b) Implicit differentiation.

Example 2: Find $\frac{dy}{dx}$ for $xy = 4$.

Example 3: Find $\frac{dy}{dx}$ for the equation $x^3y - 2x^2y^3 + x^2 - 3 = 0$.

Example 4: Find $\frac{dy}{dx}$ for the equation $\frac{1}{y^4} - \frac{5}{x^4} = 1$.

Example 5: Find $\frac{dy}{dx}$ for the equation $(x - y)^4 = y^2$.

Example 6: Find $\frac{dy}{dx}$ for the equation $x + \cos(x^2 y) = y$

Finding higher-order derivatives using implicit differentiation:

To find the second derivative, denoted $\frac{d^2y}{dx^2}$, differentiate the first derivative $\frac{dy}{dx}$ with respect to x .

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Example 7: Find $\frac{d^2y}{dx^2}$ for the equation $x^3 - 2x^2 = y$.

Example 8: Find $\frac{d^2y}{dx^2}$ for the equation $xy^2 - y = 3$.

Example 9: Find the equation of the tangent line to the ellipse $4x^2 + 16y^2 = 64$ at the point $(2, \sqrt{3})$.

Definition: Two curves are said to be *orthogonal* if at each point of intersection, their tangent lines are perpendicular.

Example 10: Show that the hyperbola $x^2 - y^2 = 5$ and the ellipse $4x^2 + 9y^2 = 72$ are orthogonal.