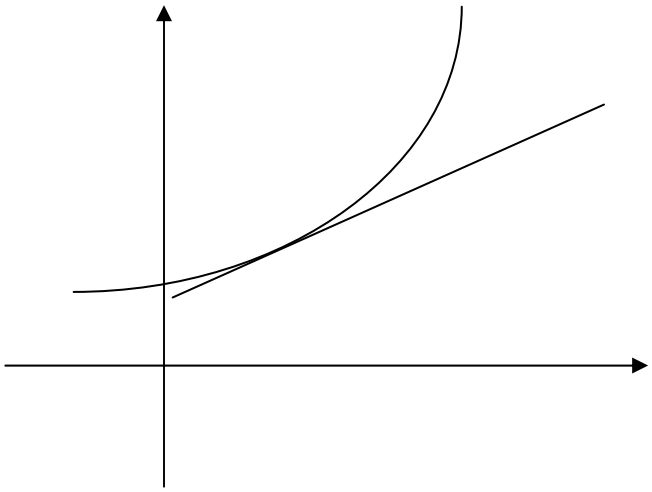
3.9: Differentials

Differentials:



If $y = f(x)$ is a differentiable function, we can let dx represent an amount of change in x.

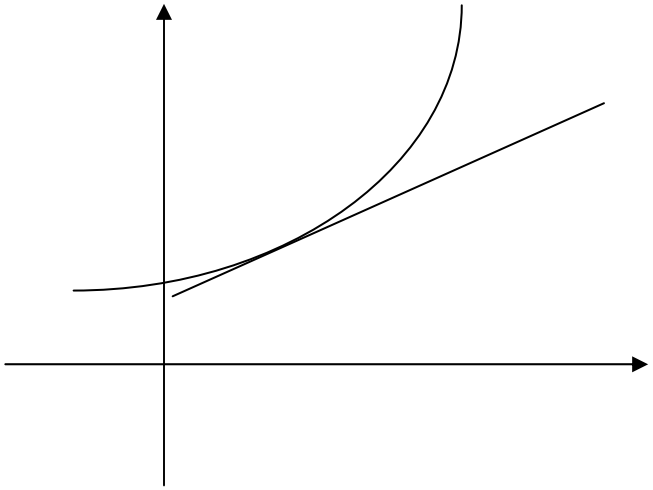
Then the *differential dy* is defined to be $dy = f'(x)dx$.

dy is an approximation to $\Delta y = f(x+\Delta x) - f(x)$, which is the actual change in y.

Example 1: Find the differential dy for $y = \sqrt{6+x}$. Evaluate it when $x = 10$ and $dx = -0.3$. Compare it to the exact value of Δy.

Example 2: Compute dy and Δy if $y = 5x + x^3$ as x changes from 3 to 3.05.

The linearization of a function:



$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) \approx \frac{f(x) - f(a)}{x - a}$$

$$f(x) - f(a) \approx f'(a)(x - a)$$

$$f(x) \approx f(a) + f'(a)(x - a)$$

This tangent line gives us an approximation for the value of the function near *a*.

The *linearization* of *f* at *a* is $L(x) = f(a) + f'(a)(x - a)$ The approximation $f(x) \approx L(x)$ is the standard linear approximation of *f* at *a*.

Example 3: Find the linearization of $f(x) = x^4$ at 3. Use this to approximate $(3.013)^4$ and $(2.999)^4$.

Example 4: Find the linearization of $f(x) = \sin x$ at $60°$. Use this to approximate $\sin 62°$ and $\sin 58°$.

Error propagation:

If x is the measured value of a variable and $x + \Delta x$ is the exact value of the variable, then Δx is the measurement error. If we use the measured value of x to calculate the value of a function f, then the propagated error is $\Delta y = f(x + \Delta x) - f(x)$. The propagated error can be estimated by calculating $dy = f'(x)dx \approx f'(x)\Delta x$.

Estimating propagated error:

If x is the measured value of a variable and Δx is the measurement error, then:

Estimated propagated error: $dy = f'(x)dx \approx f'(x)\Delta x$

Estimated relative error: $\frac{dy}{y}$

Example 5: The measurement of the radius of a circle is 20 inches with a maximum error of 0.10 inch. Approximate the maximum propagated error and the relative error in computing the area and the circumference of the circle.

Example 6: The measurements of the height and inside radius of a right cylinder are 50 feet and 30 feet, respectively. The maximum possible error in each measurement is about 3 inches per each 10 feet of measured length. Approximate the maximum propagated error and the relative error in computing the volume of the cylinder.