4.1: Antiderivatives and Indefinite Integration

Definition: An *antiderivative* of f is a function whose derivative is f.

i.e. A function F is an antiderivative of f if $F'(x) = f(x)$.

Example 1: ______________________ is an antiderivative of $f(x) = 3x^2 + 5$.

What are some more antiderivatives of $f(x) = 3x^2 + 5$?

So we have a whole "family" of antiderivatives of f.

Definition: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Theorem: If F is an antiderivative of f on an interval I, then all antiderivatives of f on I will be of the form

$$F(x) + C$$

where C is an arbitrary constant.

Example 2: Find the general form of the antiderivatives of $f(x) = 3x^2 + 5$.

Example 3: Find the general form of the antiderivatives of $f(x) = 6x^5 + \cos x$.

Integration:

Integration is the process of finding antiderivatives.

$\int f(x)dx$ is called the *indefinite integral* of *f*.

$\int f(x)dx$ is the family of antiderivatives, or the most general antiderivative of *f*.

This means: $\int f(x)dx = F(x) + c$, where $F'(x) = f(x)$.

The c is called the *constant of integration*.

Integral sign

"with respect to x"

$$\int f(x)dx$$

Integrand

Example 4: Find $\int 3x^2 + 5\, dx$.

Example 5: Find $\int 6x^5 + \cos x\, dx$.

Example 6: Find $\int \sec^2 dx$.

Rules for Finding Antiderivatives:

Notation in this table: F is an antiderivative of f, G is an antiderivative of g,

Function	Antiderivative
k	$kx+c$
$kf(x)$	$kF(x)$
$f(x)+g(x)$	$F(x)+G(x)$
x^n for $n \neq -1$	$\dfrac{x^{n+1}}{n+1}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\sec^2 x$	$\tan x$
$\sec x \tan x$	$\sec x$

1. $\int k\ dx = kx + c$ (k a constant)

2. $\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$

3. $\int k\ f(x)\ dx = k\int f(x)\ dx$ (k a constant)

4. $\int [f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$

5. $\int \cos x\, dx = \sin x + c$

6. $\int \sin x\, dx = -\cos x + c$

7. $\int \sec^2 x\, dx = \tan x + c$

8. $\int \sec x \tan x\, dx = \sec x + c$

Example 7: Find the general antiderivative of $f(x) = \dfrac{1}{2}$.

<u>Example 8:</u> Find $\int x^3\,dx$.

<u>Example 9:</u> Find $\int 7x^2\,dx$.

<u>Example 10:</u> Find $\int \frac{1}{x^5}\,dx$.

<u>Example 11:</u> Find the general antiderivative of $f(x) = \frac{5}{x^2}$.

<u>Example 12:</u> $\int \left(6x^2 - 3x + 9\right)dx$

<u>Example 13:</u> Find $\int 3\sqrt{x}\,dx$.

Example 14: Find $\int (3\cos x + 5\sin x)\, dx$.

Example 15: Find $\int \frac{x^7 - \sqrt[3]{x} + 3x^2}{x^5}\, dx$.

Example 16: Find the general antiderivative of $f(\theta) = \frac{\sin\theta}{3}$.

Example 17: $\int \left(\sqrt[3]{x} + \frac{2}{\sqrt{x}} \right) dx$

Example 18: $\int (6y^2 - 2)(8y + 5)\, dy$

Differential equations:

A *differential equation* is an equation involving the derivative of a function. To solve a differential equation means to find the original function.

An *initial value problem* is a common type of differential equation in which a derivative and an initial condition are given.

<u>Example 19:</u> Given $f'(x) = x^2 - 7$, find *f*. This is an example of a differential equation.

<u>Example 20:</u> Suppose that $f'(x) = 3x^2 + 2\cos x$ and $f(0) = 3$. Find $f(x)$.

Example 21: Suppose that $f''(x) = 2x^3 - 6x^2 + 6x$, $f'(2) = -1$, and $f(-1) = 4$. Find $f(x)$.

Example 22: Suppose that $f''(x) = 12x^2 - 18x$, $f(1) = 2$, and $f(-3) = 1$. Find $f(x)$.

Velocity and acceleration (rectilinear motion):

We already know that if $f(t)$ is the position of an object at time *t*, then $f'(t)$ is its velocity and $f''(t)$ is its acceleration.

<u>Note</u>: Acceleration due to gravity near the earth's surface is approximately 9.8 m/s^2 or 32 ft/s^2.

<u>Example 23:</u> Suppose a particle's velocity is given by $v(t) = 2\sin t + \cos t$ and its initial position is $s(0) = 3$. Find the position function of the particle.

<u>Example 24:</u> Suppose a ball is thrown upward from a 30-foot bridge over a river at an initial velocity of 40 feet per second. How high does it go? When does it hit the water?