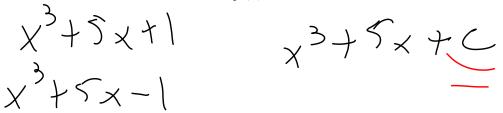
# 4.1: Antiderivatives and Indefinite Integration

<u>Definition</u>: An *antiderivative* of f is a function whose derivative is f.

i.e. A function F is an antiderivative of f if F'(x) = f(x).

Example 1:  $\searrow$  5  $\swarrow$  is an antiderivative of  $f(x) = 3x^2 + 5$ .

What are some more antiderivatives of  $f(x) = 3x^2 + 5$ ?



So we have a whole "family" of antiderivatives of f.

<u>Definition</u>: A function F is called an antiderivative of f on an interval I if F'(x) = f(x) for all x in I.

Theorem: If F is an antiderivative of f on an interval I, then all antiderivatives of f on I will be of the form

$$F(x) + C$$

where C is an arbitrary constant.

**Example 2:** Find the general form of the antiderivatives of  $f(x) = 3x^2 + 5$ .

**Example 3:** Find the general form of the antiderivatives of  $f(x) = 6x^5 + \cos x$ .

# **Integration:**

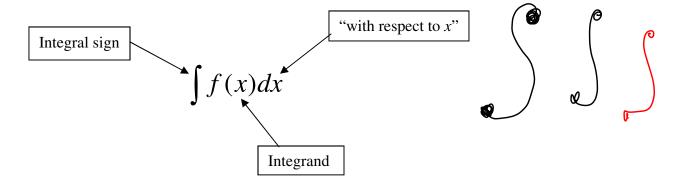
<u>Integration</u> is the process of finding antiderivatives.

 $\int f(x)dx$  is called the *indefinite integral* of f.

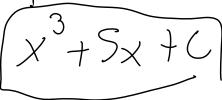
 $\int f(x)dx$  is the family of antiderivatives, or the most general antiderivative of f.

This means:  $\int f(x)dx = F(x) + c$ , where F'(x) = f(x).

The c is called the *constant of integration*.



**Example 4:** Find  $\int 3x^2 + 5 dx$ 

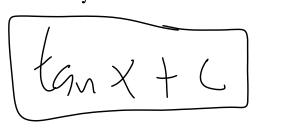


**Example 5:** Find  $\int 6x^5 + \cos x \, dx$ .



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**Example 6:** Find  $\int \sec^2 dx$ .



### Rules for Finding Antiderivatives:

Notation in this table: F is an antiderivative of f, G is an antiderivative of g,

	Function	Antiderivative
	k	kx + c
	kf(x)	kF(x)
	f(x) + g(x)	F(x) + G(x)
<u></u>	$x^n$ for $n \neq -1$	$\chi^{n+1}$
		$\frac{n+1}{n+1}$
	$\cos x$	$\sin x$
	$\sin x$	$-\cos x$
	$\sec^2 x$	tan x
	sec x tan x	sec x

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1. 
$$\int k \ dx = kx + c$$
 (k a constant)

$$\int X dx = \frac{1}{5} + C$$

2. 
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$$

3. 
$$\int k f(x) dx = k \int f(x) dx$$
 (k a constant)

4. 
$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$5. \int \cos x \, dx = \sin x + c$$

$$6. \int \sin x \, dx = -\cos x + c$$

$$7. \int \sec^2 x \, dx = \tan x + c$$

8. 
$$\int \sec x \tan x \, dx = \sec x + c$$

**Example 7:** Find the general antiderivative of  $f(x) = \frac{1}{2}$ .

 $F(x) = \frac{1}{2}x + C$ 

**Example 8:** Find  $\int x^3 dx$ .

**Example 9:** Find  $\int 7x^2 dx$ .

**Example 10:** Find  $\int \frac{1}{x^5} dx$ .

 $\int_{-4}^{6} \frac{1}{x^{4}} dx = \frac{1}{4x^{4}} + C$ 

**Example 11:** Find the general antiderivative of  $f(x) = \frac{5}{x^2}$ 

 $\int 5x^{-2} dx = \int x + 2 = -5 + 2$ 

**Example 12:**  $\int (6x^2 - 3x + 9) dx$ 

 $\frac{6x^3-3x^2+9x+2}{2}=2x^3-\frac{3}{2}x^2$ 

**Example 13:** Find  $\int 3\sqrt{x} \, dx$ .

53×2/2 = 23×4c = 2x+c

**Example 14:** Find 
$$\int (3\cos x + 5\sin x) dx$$
.

$$\frac{1}{3} - 5 = \frac{1}{3} - \frac{15}{3} = \frac{4.1.5}{14/3}$$

Example 15: Find 
$$\int \frac{x^7 - \sqrt[3]{x} + 3x^2}{x^5 + \sqrt[3]{3}} dx$$
.  

$$= \int x^7 - x^3 + 3x^2 dx$$

**Example 16:** Find the general antiderivative of  $f(\theta) = \frac{\sin \theta}{2}$ 

$$-\frac{1}{3}\cos\theta + C$$

Example 18: 
$$\int (6y^2 - 2)(8y + 5) dy$$

Example 18: 
$$\int (6y^2-2)(8y+5)dy$$

$$\int (48y^3+30y^2-16y-10)dy$$

$$\frac{12y^4+16y^3-6y^2-10y+1}{2}$$

#### **Differential equations:**

A *differential equation* is an equation involving the derivative of a function. To solve a differential equation means to find the original function.

An *initial value problem* is a common type of differential equation in which a derivative and an initial condition are given.

**Example 19:** Given  $f'(x) = x^2 - 7$ , find f. This is an example of a differential equation.

$$\int_{3}^{2} \sqrt{-1} dx = \frac{3}{3} - 1x + C$$

Example 20: Suppose that  $f'(x) = 3x^2 + 2\cos x$  and f(0) = 3. Find f(x).  $\int (3x^2 + 2\cos x) dx = x^3 + 2\sin x + C$   $\int (3x^2 + 2\cos x) dx = x^3 + 2\cos x + C$   $\int (3x^$ 

**Example 21:** Suppose that  $f''(x) = 2x^3 - 6x^2 + 6x$ , f'(2) = -1, and f(-1) = 4. Find f(x).

$$S(2x^{3}-6x^{2}+6x) dx$$

$$S(2x^{3}-6x^{2}+6x)$$

**Example 22:** Suppose that  $f''(x) = 12x^2 - 18x$ , f(1) = 2, and f(-3) = 1. Find f(x).

$$\int 12x^{2}-18x dx$$

$$4x^{3}-9x^{2}+C=F(x)$$

$$4x^{3}-9x^{2}+C=F(x)$$

$$4(1)^{3}-7(1)^{2}+C=2$$

$$4(1)^{3}-7($$

### **Velocity and acceleration (rectilinear motion):**

We already know that if f(t) is the position of an object at time t, then f'(t) is its velocity and f''(t) is its acceleration.

Note: Acceleration due to gravity near the earth's surface is approximately 9.8 m/s<sup>2</sup> or 32 ft/s<sup>2</sup>.

**Example 23:** Suppose a particle's velocity is given by  $v(t) = 2\sin t + \cos t$  and its initial position is s(0) = 3. Find the position function of the particle.

$$Sv(t)At=4t)+C$$
  $3=-2cos(a)+5m(a)+C$   $S(2sint+cost)At$   $s=c$   $-2cost+5int+c$   $-2cost+5int+f$ 

**Example 24:** Suppose a ball is thrown upward from a 30-foot bridge over a river at an initial velocity of 40 feet per second. How high does it go? When does it hit the water?

$$a(t) = -32$$
 $v(t) = -32t + 40$ 
 $s = -32t + 40$ 
 $v(t) = -32t + 40$ 
 $v$ 

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