

5.2: The Natural Logarithmic Function: Integration

Using the derivative of the natural logarithmic function to obtain an antiderivative:

Example 1: Find the derivative of $g(x) = \ln|x|$.

Note that $f(x) = \ln x$ has the same derivative as $g(x) = \ln|x|$.

Therefore $\frac{d}{dx} \ln|x| = \frac{1}{x}$. This means that $f(x) = \ln|x|$ is an antiderivative of $F(x) = \frac{1}{x}$.

$$\int \frac{1}{x} dx = \ln|x| + c$$

Recall: The power rule for integrals $\int x^n dx = \frac{x^{n+1}}{n+1}$ had a restriction: $n \neq -1$. Now we can handle this case.

Example 2: Determine $\int \frac{x^2}{x^3 + 4} dx$.

Example 3: Determine $\int \frac{7}{2 - 5x} dx$.

Example 4: Determine $\int_2^5 \frac{1}{3x} dx$.

Example 5: Determine $\int \frac{x^7 - x + 3x^4}{x^5} dx$.

Example 6: Find $\int \frac{(\ln x)^4}{x} dx$.

Example 7: Find $\int \frac{\ln(3x)}{x} dx$.

Example 8: Find $\int \frac{x}{x^2 - 8} dx$.

Example 9: Find $\int \frac{4x^2 - 5x - 12}{x^2 - 3} dx$.

Example 10: Find $\int \frac{4x^2 - 7x + 1}{2x - 3} dx$.

Integrating the remaining trigonometric functions:

Example 11: Determine $\int \tan x \, dx$.

Example 12: Determine $\int \cot x \, dx$.

Example 13: Determine $\int \sec x \, dx$.

Example 14: Determine $\int \csc x \, dx$.