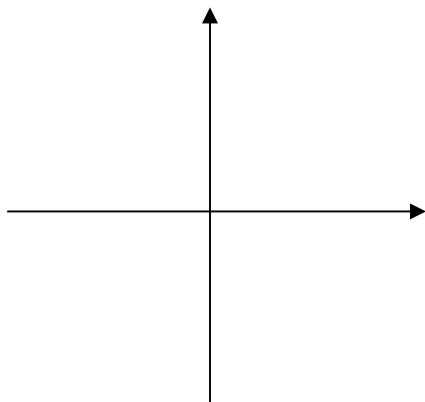


## **5.4: Exponential Functions: Differentiation and Integration**

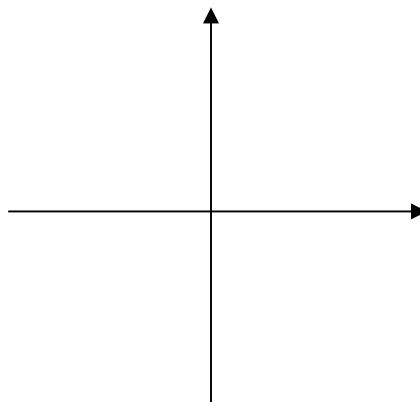
### **Short Review:**

An *exponential* function takes the form  $f(x) = b^x$ , where  $b > 0$  and  $b \neq 1$ .

For any exponential function  $f(x) = b^x$ , the graph looks like one of the following.



$b > 1$



$0 < b < 1$

### Notice:

- Domain is \_\_\_\_\_.
- Range is \_\_\_\_\_.
- Horizontal asymptote is \_\_\_\_\_.
- Always passes through the points \_\_\_\_\_.

### **The natural exponential function:**

The number  $e$  can be defined in several ways.

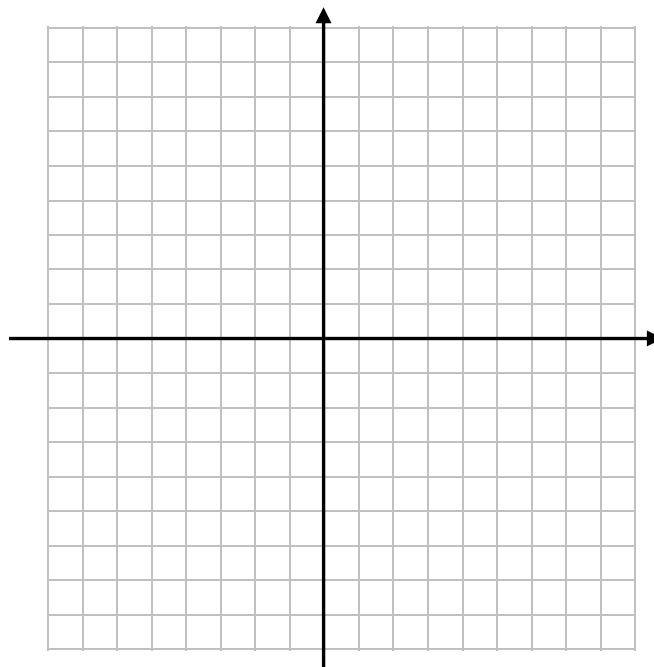
#### One definition of the number $e$ :

$$e \text{ is the number such that } \lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$$

$$e \approx 2.718281828459$$

The slope of the tangent line at the point  $(0,1)$  is equal to 1.

**The graph of  $f(x) = e^x$ :**



Another definition of the number  $e$ :

$$e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x \quad \text{or, equivalently,} \quad e = \lim_{x \rightarrow 0} (1+x)^{1/x}$$

**Derivatives of exponential functions:**

$$\frac{d}{dx}(e^x) = e^x$$

**Example 1:** Find the derivative of  $f(x) = -7e^x$ .

**Example 2:** Find the derivative of  $f(x) = 5\sqrt{e^x + 7}$ .

**Example 3:** Find the derivative of  $f(x) = e^x \sin x$ .

**Example 4:** Find the derivative of  $g(x) = e^{-7x} + 2x^3 - 4e$ .

**Example 5:** Find the derivative of  $y = e^{x^2 + 4x}$ .

**Example 6:** Find the derivative of  $f(x) = \cos(e^x - x)$ .

**Example 7:** Find the equation of the tangent line to the graph of  $f(x) = (e^x + 2)^2$  at the point  $(0, 9)$ .

**Integration of exponential functions:**

$$\int e^x dx = e^x + c$$

**Example 8:** Determine  $\int (x^2 - 5e^x) dx$

**Example 9:** Find  $\int e^{5t} dt$ .

**Example 10:** Find  $\int_1^3 e^{2x-3} dx$ .

**Example 11:** Find  $\int te^{t^2} dt$ .

**Example 12:** Determine  $\int \frac{e^x}{\sqrt[3]{e^x + 1}} dx$ .

**Example 13:** Determine  $\int \frac{e^x - e^{-x}}{e^{3x}} dx$