

5.7: Inverse Trigonometric Functions – Integration

Two important integration rules come from the inverse trigonometric differentiation rules.

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Example 1: Find $\int \frac{1}{\sqrt{1-9x^2}} dx$.

Example 2: Find $\int \frac{1}{x^2+7} dx$.

More general forms of these integration rules are

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C.$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C$$

Example 3: Find $\int \frac{4x}{x^4 + 25} dx$.

Another antiderivative:

$$\int \frac{1}{x\sqrt{x^2 - 1}} dx = \sec^{-1}|x| + C$$

Example 4: Find $\int \frac{1}{x\sqrt{x^2 - 4}} dx$.

Example 5: Find $\int \frac{1}{4x^2 - 12x + 17} dx$.

Example 6: Find $\int \frac{x-7}{\sqrt{5-4x^2}} dx$.