7.1: Area of a Region Between Two Curves

Because the definite integral represents the "net" area under a curve, we can use integration to find the area between curves.

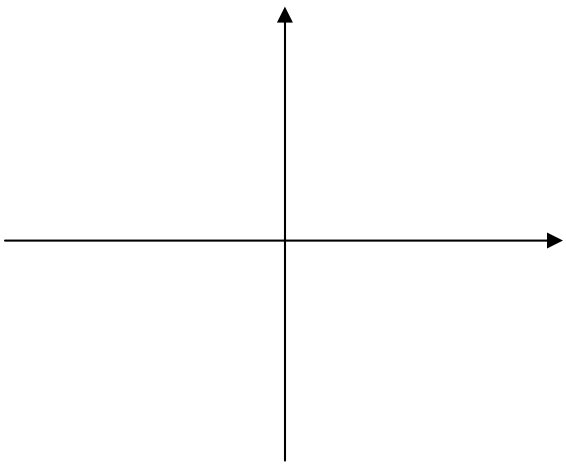
If *f* and *g* are continuous and $f(x) \geq g(x)$ on $[a,b]$, then the area between $y = f(x)$, $y = g(x)$, and the lines $x = a$ and $x = b$ is given by

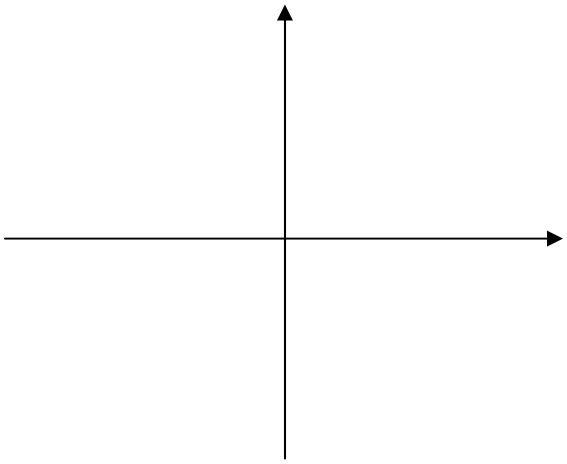
$$\text{Area} = \int_a^b [f(x) - g(x)]dx$$

Why?

Note: If $g(x) \geq f(x)$ on $[a,b]$, then $\int_a^b [f(x) - g(x)]dx$ is negative.

Some different types of area scenarios:





Example 1: Find the area of the region bounded by $f(x) = x^3 - 1$ and the lines $y = 0$, $x = -2$, and $x = 4$.

Example 2: Find the area of the region bounded by $y = \cos x$, $y = \sin x$, and the lines $x = 0$, and $x = 2\pi$.

Example 3: Find the area of the region completely enclosed by the graphs of $y = x^2 + 1$ and $y = 2x + 9$.

Example 4: Find the area of the region completely enclosed by the graphs of $y = x^3$ and $y = x$.

Example 5: Find the area of the region completely enclosed by the graphs of $x = y^2$ and $x = 4$.

Example 6: Find the area of the region completely enclosed by the graphs of $x = 3 - y^2$ and $x = y + 1$.