2.7: Linear Inequalities

Note Title 1/29/2015

Additive Property of Inequality

If $A < B$, then $A + C < B + C$.

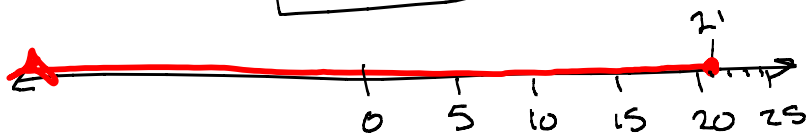
This means that we can add (or subtract) the same expression on both sides.

Example: Solve $x - 8 \leq 13$. Graph your solution.

$+8 \quad +8$

$x \leq 13 + 8$

$\boxed{x \leq 21}$



Could graph it like this:

Multiplicative Property of Inequality

If $A < B$ and $C > 0$, then $AC < BC$.

If $A < B$ and $C < 0$, then $AC > BC$.

This means:

We can multiply (or divide) both sides by the same positive number.

We can multipty (or divide) both sides by the same negative number if we reverse the inequality sign.

Why? $1 < 2$ True.

multiply both sides by 2: $1(2) < 2(2)$

$2 < 4$ True.

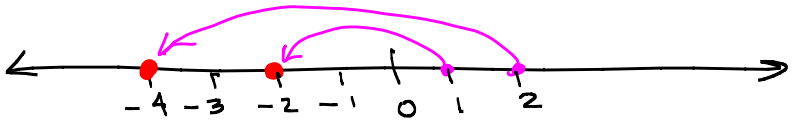
Start with $1 < 2$ again. (still true)

multiply both sides by -2:

$1 < 2$ True

$1(-2) < 2(-2)$ False

$-2 < -4$ False!



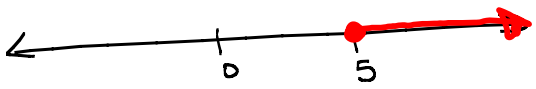
multiplying by a negative changed the order.

Example: Solve and graph.

$$4x \geq 20$$

$$\frac{4x}{4} \geq \frac{20}{4}$$

$$\boxed{x \geq 5}$$



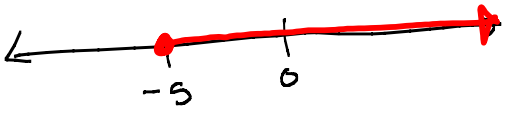
Ex: Solve and graph.

$$4x \geq -20$$

$$\frac{4x}{4} \geq \frac{-20}{4}$$

$$\boxed{x \geq -5}$$

(we're dividing by a positive, so we don't reverse the sign)

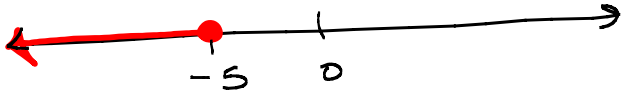


Ex: Solve and graph.

$$-4x \geq 20$$

$$\frac{-4x}{-4} \leq \frac{20}{-4}$$ ← We are dividing by a negative, so we reverse the inequality sign

$$\boxed{x \leq -5}$$

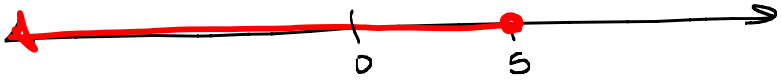


Ex: Solve and graph.

$$-4x \geq -20$$

$$\frac{-4x}{-4} \leq \frac{-20}{-4}$$ ← Reverse the inequality sign! Because we are dividing by a negative.

$$\boxed{x \leq 5}$$

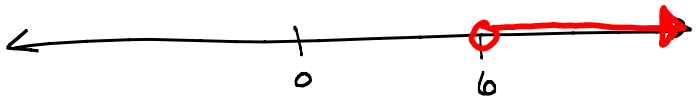


Ex: Solve and graph.

Method 1: (Putting variables on left)

$$3x + 7 < 5x - 5$$
$$-5x \quad\quad -5x$$
$$3x - 5x + 7 < -5$$
$$-2x + 7 < -5$$
$$-7 \quad\quad -7$$
$$-2x < -5 - 7$$
$$-2x < -12$$
$$\frac{-2x}{-2} > \frac{-12}{-2}$$ ← Reverse the inequality sign!

$$\boxed{x > 6}$$



Previous examples worked another way:

Method #2
(Variables on right side)

$$3x+7 < 5x-5$$
$$\begin{array}{r} -3x \\ \end{array} \qquad \begin{array}{r} -3x \\ \end{array}$$
$$7 < 5x-3x-5$$
$$7 < 2x-5$$
$$\begin{array}{r} +5 \\ \end{array} \qquad \begin{array}{r} +5 \\ \end{array}$$
$$7+5 < 2x$$
$$12 < 2x$$
$$\frac{12}{2} < \frac{2x}{2}$$
$$6 < x$$

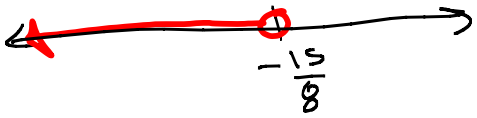
same as $\boxed{x > 6}$

Ex: Solve and graph.

$$-\frac{2}{3}x > \frac{5}{4}$$
$$\frac{-\frac{2}{3}x}{-\frac{2}{3}} < \frac{\frac{5}{4}}{-\frac{2}{3}}$$

← Reverse the inequality sign

$$\left(-\frac{3}{2}\right)\left(-\frac{2}{3}x\right) < \frac{5}{4}\left(-\frac{3}{2}\right)$$
$$1x < -\frac{15}{8}$$
$$x < -\frac{15}{8}$$
$$\boxed{x < -1\frac{7}{8}}$$



Combined inequalities

Inequalities can be combined with AND or OR.

Examples: Solve and graph.

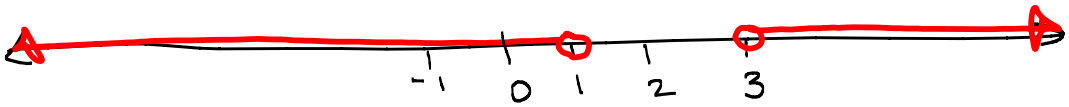
$2x + 1 < 3$ OR $2x + 1 > 7$

$-1 \quad -1 \qquad -1 \quad -1$

$2x < 2$ $\qquad 2x > 6$

$\frac{2x}{2} < \frac{2}{2}$ $\qquad \frac{2x}{2} > \frac{6}{2}$

$x < 1$ OR $x > 3$ solution



Ex: Solve and graph.

$$-6 \leq 2x + 1 < 17$$

This means:

$-6 \leq 2x + 1$ AND $2x + 1 < 17$

$-1 \quad -1 \qquad -1 \quad -1$

$-7 \leq 2x$ $\qquad 2x < 16$

$\frac{-7}{2} \leq \frac{2x}{2}$ $\qquad \frac{2x}{2} < \frac{16}{2}$

$x < 8$

Rewrite: $x \geq -3\frac{1}{2}$

$\{ -3\frac{1}{2} \leq x$ AND

Solution is the overlapping part

Final Solution: should combine:

$$-3\tfrac{1}{2} \leq x < 8$$

See next page

Method #2:

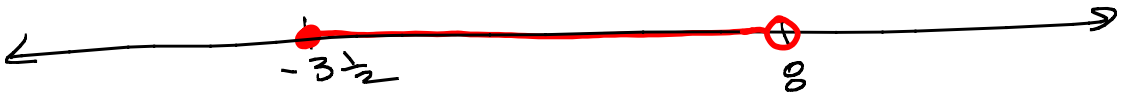
$-6 \leq 2x+1 < 17$

$-1 \quad -1 \quad -1$ ← subtract 1 from all 3 "sides"

$-7 \leq 2x < 16$

$\frac{-7}{2} \leq \frac{2x}{2} < \frac{16}{2}$ ← Divide all 3 "sides" by 2

$\boxed{-3\frac{1}{2} \leq x < 8}$



Note: $10 \leq x < 8$ is impossible.

(x cannot be both greater than 10 and also smaller than 8)

Ex: Solve $-8 \leq 6-2x < 5$

$-6 \quad -6 \quad -6$

$-14 \leq -2x < -1$

Reverse all the inequality signs → $\frac{-14}{-2} \geqslant \frac{-2x}{-2} > \frac{-1}{-2}$

$7 \geqslant x > \frac{1}{2}$

Rewrite: $\boxed{\frac{1}{2} < x \leq 7}$ solution