3.4: Graphing Lines using Slope (Continued)

Note Title 2/5/2015

Recall:

Definition of slope:

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$, where (x_1, y_1) and (x_2, y_2) are points on the line.

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

Example: Find the slope of the line containing the points $(-3, 2)$ and $(5, 8)$.

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{5-(-3)} = \frac{6}{5+3} = \frac{6}{8} = \boxed{\frac{3}{4}}$$

$x_1 = -3$, $y_1 = 2$, $x_2 = 5$, $y_2 = 8$

Note: We would get the same answer if we reversed the order of the 2 points.

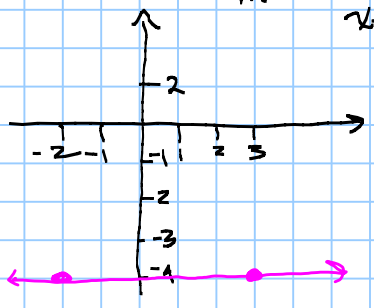
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-8}{-3-5} = \frac{-6}{-8} = \boxed{\frac{3}{4}}$$

Ex: Find the slope of the line containing the points $(4, -6)$ and $(-1, -3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-6)}{-1-4} = \frac{-3+6}{-5} = \frac{3}{-5} = \boxed{-\frac{3}{5}}$$

Example: Find the slope of the line containing the points $(3, -4)$ and $(-2, -4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-4)}{-2-3} = \frac{-4+4}{-5} = \frac{0}{-5} = \boxed{0}$$



This line is horizontal.

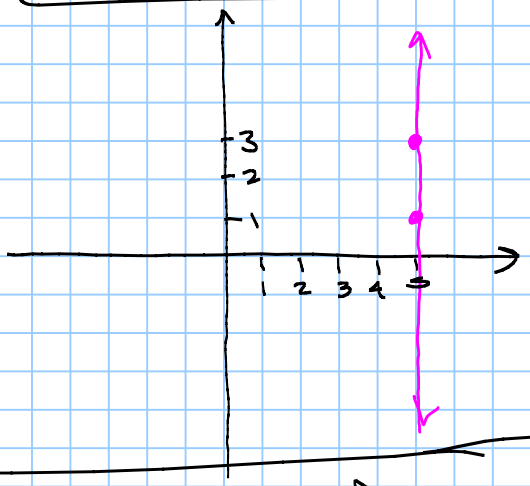
The slope of every horizontal line is 0.

Example: Find the slope of the line containing the points $(5, 1)$ and $(5, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{5-5} = \frac{2}{0}$$

Undefined!
Cannot divide by 0.

The slope of this line is undefined.



This is a vertical line

Every vertical line has undefined slope.

Slopes of horizontal and vertical lines: ✱

Important:
- ✱ A horizontal line has slope 0.
- ✱ A vertical line has undefined slope.

3.5: Equations of Lines

Slope-intercept form of a line:

$y = mx + b$, where m = slope and b is the y-intercept.

Graph the line $y = 2x - 3$ using the slope and y-intercept.

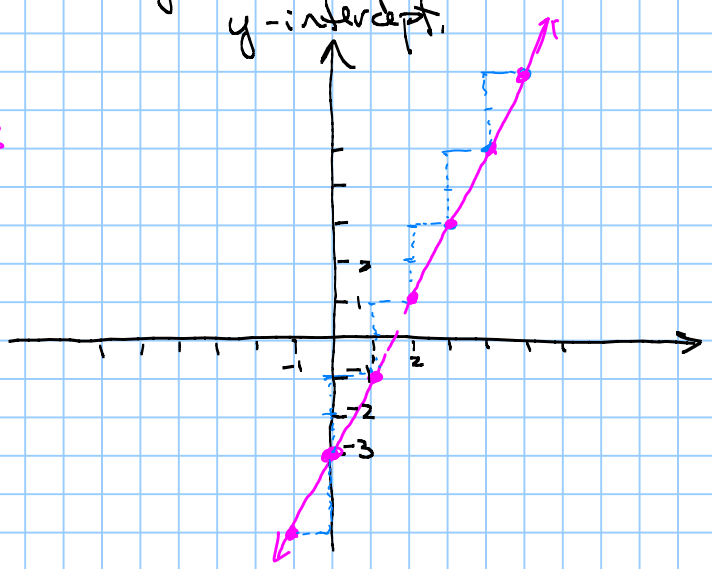
$y = mx + b$
so $m = 2$, $b = -3$

slope
$m = 2 = +\frac{2}{1}$

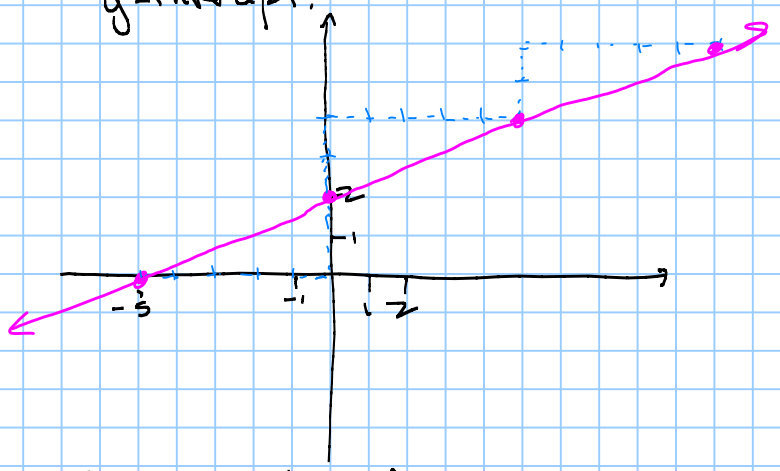
vertical change = rise = 2
horizontal change = run = 1

y-intercept

$b = -3$
so plot the point $(0, -3)$



Example: Graph the line $y = \frac{2}{5}x + 2$ using slope and y-intercept.



Slope: $m = +\frac{2}{5}$

rise = 2
run = 5

y-intercept: $b = 2$

Example: Graph the line $4x + 3y = 9$ by writing it in slope-intercept form.

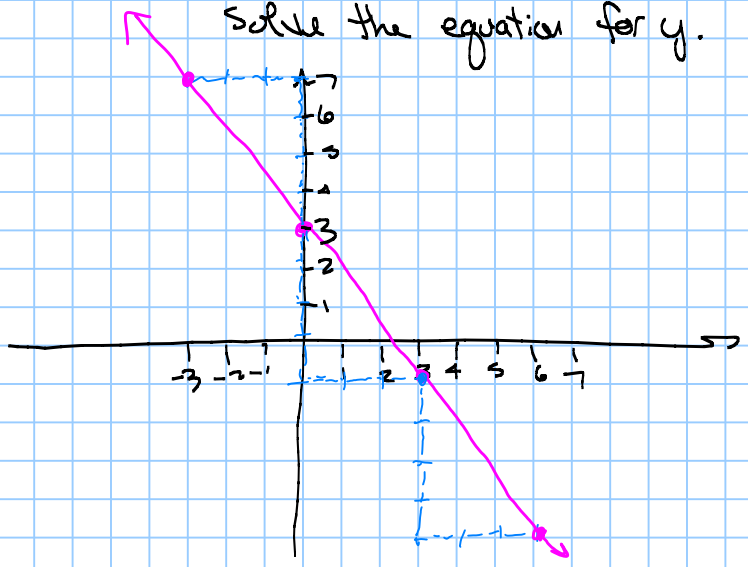
Write in slope-intercept form ($y = mx + b$):

Solve the equation for y.

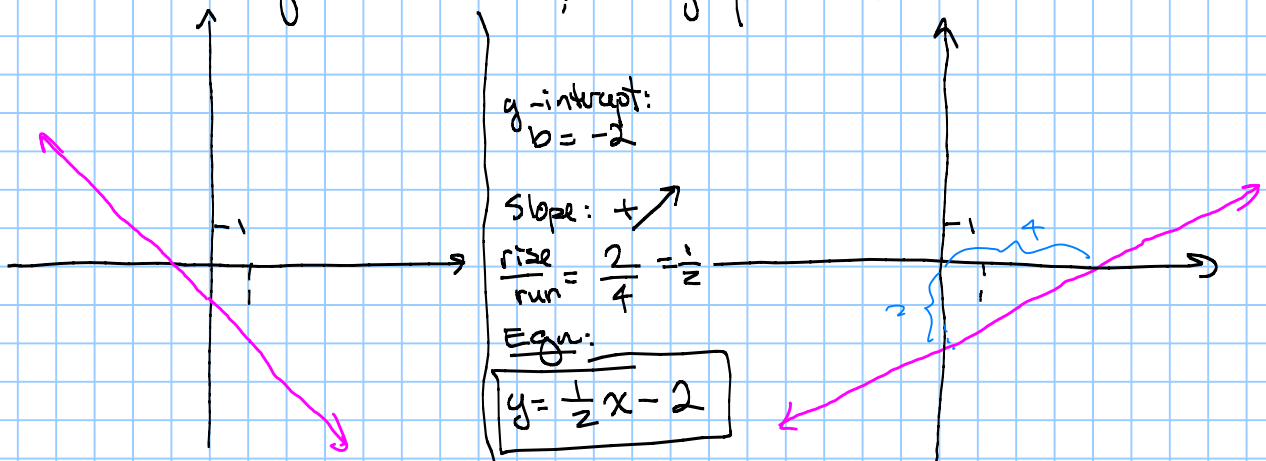
$$4x + 3y = 9$$
$$-4x \quad\quad -4x$$
$$3y = -4x + 9$$
$$\frac{3y}{3} = -\frac{4x}{3} + \frac{9}{3}$$
$$y = -\frac{4}{3}x + 3$$

$m = -\frac{4}{3}$

y-intercept: $b = 3$



Ex: Write the equation of the graphed lines.



y-intercept: $b = -1$

slope: $\searrow$ m must be negative

$\frac{\text{rise}}{\text{run}} = \frac{2}{2} = 1$ or $\frac{2}{2} = 1$ or $\frac{3}{3} = 1$ } $m = -1$

So eqn is $y = -1x - 1$ or $y = -x - 1$

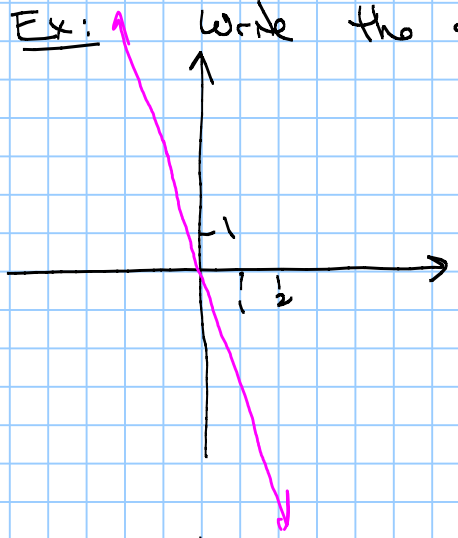
y-intercept: $b = -2$

slope: $\nearrow$

$\frac{\text{rise}}{\text{run}} = \frac{2}{4} = \frac{1}{2}$

Eqn: $y = \frac{1}{2}x - 2$

Ex: Write the equation of the graphed lines.



y-intercept: $b = 0$

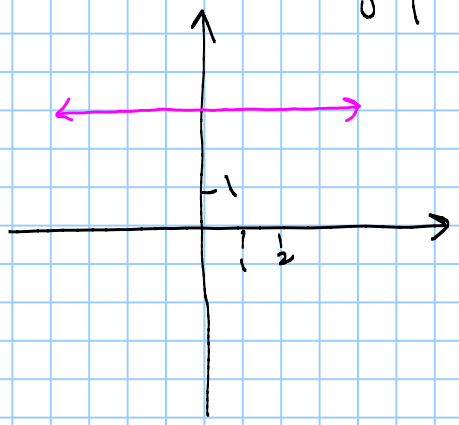
Slope: $-$

$\frac{\text{rise}}{\text{run}} = \frac{3}{1}$ (it's negative)

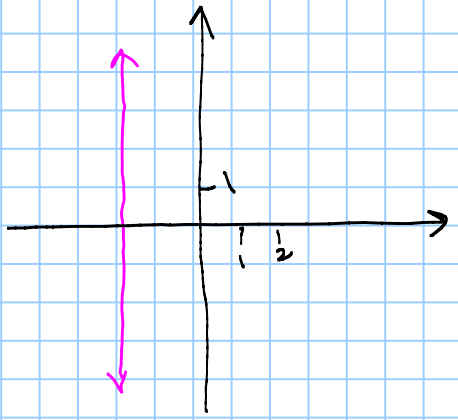
So $m = -\frac{3}{1} = -3$

Eqn: $y = -3x + 0$

$\boxed{y = -3x}$



Slope = 0
(it's horizontal)
y-intercept: 3
Could write $y = 0x + 3$
eqn of line $\boxed{y = 3}$



Eqn of line: $\boxed{x = -2}$

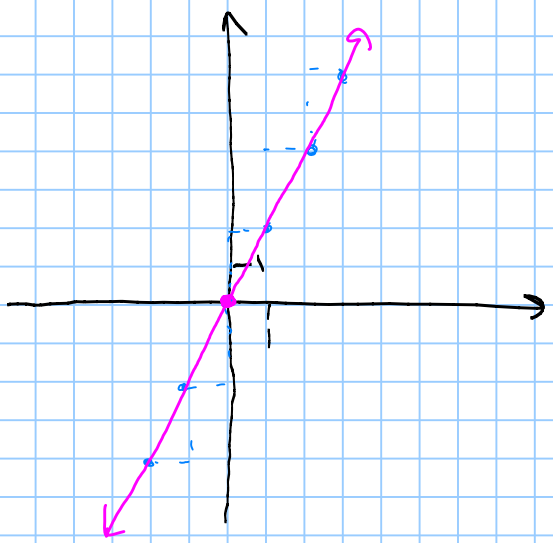
Note: the slope is undefined

Ex:
Graph the equation.

$y = 2x$

x	$y = 2x$	
0	$2(0) = 0$	$(0,0)$
1	$2(1) = 2$	$(1,2)$
-2	$2(-2) = -4$	$(-2,-4)$

One way to do it: plot points.



OR, use $y = mx + b$

$y = 2x$

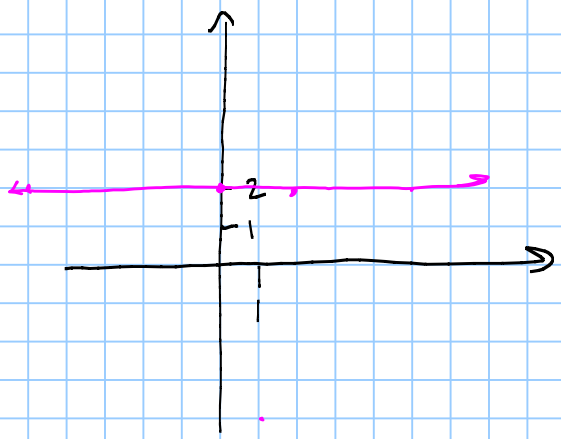
$y = 2x + 0$

y-intercept: 0

Slope: $+\frac{2}{1}$ ↗

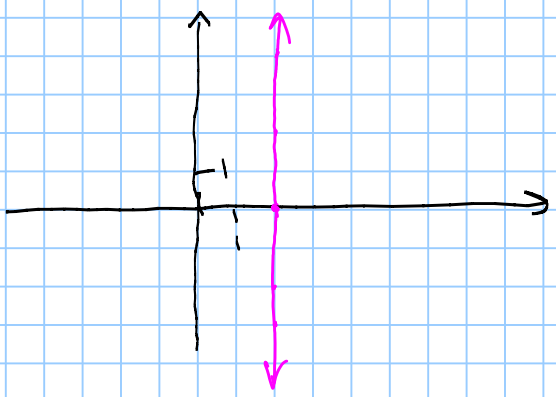
Ex: Graph the equation.

$y = 2$



Ex: Graph the equation

$x = 2$



Ex: Find the equation of the line containing the points $(1, -2)$ and $(3, 4)$.

Find the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{3 - 1} = \frac{4 + 2}{2} = \frac{6}{2} = 3$$

Use $y = mx + b$ with $m = 3$. Put either of the given points in for x and y.

$y = 3x + b$

using $x = 3$, $y = 4$: $4 = 3(3) + b$

$4 = 9 + b$

$-5 = b$

So my equation is: $\boxed{y = 3x - 5}$

Get same answer using $x = 1$, $y = -2$:

$y = 3x + b$

$-2 = 3(1) = b$

$-2 = 3 + b$

$-5 = b$ so eqn is $y = 3x - 5$

Point-slope form of a line:

$$y - y_1 = m(x - x_1)$$

where m = slope and (x_1, y_1) is a point on line

Use $m = 3$, $x = 3$, $y = 4$

$y - 4 = 3(x - 3)$

$y - 4 = 3x - 9$

$\boxed{y = 3x - 5}$