3.5: Equations of Lines (continued)

Note Title 2/17/2015 ~~2/5/2015~~

Example: Write the equation of the line that passes through the points $(1,-2)$ and $(3,-5)$.

$(x_1, y_1) = (1,-2)$
$(x_2, y_2) = (3,-5)$

Calculate the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-2)}{3-1} = \frac{-5+2}{2} = \frac{-3}{2} = -\frac{3}{2}$$

2 methods for getting the line:

Method #1 Use slope-intercept form:

$$y = mx + b$$

$m = -\frac{3}{2} \Rightarrow$ $y = -\frac{3}{2}x + b$ Need to find b.

Substitute either of the given points for x and y:

$x=3, y=-5 \Rightarrow$ $-5 = -\frac{3}{2}(3) + b$

$$-5 = -\frac{9}{2} + b$$

$$-5 + \frac{9}{2} = b$$

$$-\frac{10}{2} + \frac{9}{2} = b$$

$$-\frac{1}{2} = b$$

Eqn of line: $y = -\frac{3}{2}x - \frac{1}{2}$

Method 2: The Point-slope form of a line

$$y - y_1 = m(x - x_1)$$

$m = -\frac{3}{2}$, $x = 3$, $y = -5$ $\Rightarrow$

$$y - (-5) = -\frac{3}{2}(x-3)$$

$$y + 5 = -\frac{3}{2}x - \frac{3}{2}(-3)$$

$$y + 5 = -\frac{3}{2}x + \frac{9}{2}$$

$$y = -\frac{3}{2}x + \frac{9}{2} - 5 \cdot \frac{2}{2}$$

$$y = -\frac{3}{2}x + \frac{9}{2} - \frac{10}{2}$$

$\rightarrow$ $y = -\frac{3}{2}x - \frac{1}{2}$

Note: This is a rearrangement of the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

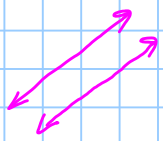
Point-slope form:

$y - y_1 = m(x - x_1)$, where m is the slope, and (x_1, y_1) is any point on the line.

Recall:

* The slope of a horizontal line is 0.
* The slope of a vertical line is undefined.

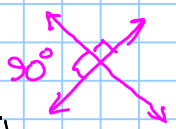
Parallel and perpendicular lines



* Parallel lines never intersect.

  Parallel lines have the same slopes.

  (The slopes of parallel lines are equal)



* Perpendicular lines intersect at 90° (right) angles

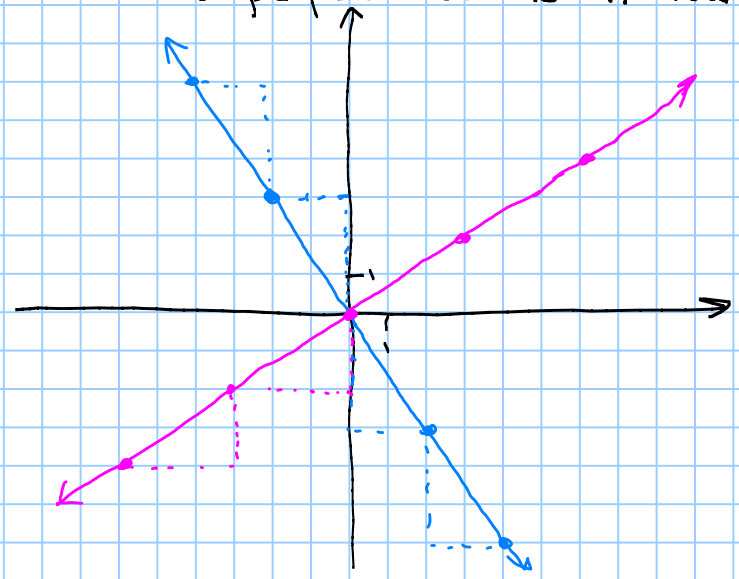
  The slopes of perpendicular lines are opposite reciprocals

  If a line has slope m, then the perpendicular line has slope $-\frac{1}{m}$.

Example: If a line has slope $\frac{2}{3}$, then

1) all lines parallel to it also have slope $\frac{2}{3}$.
2) all lines perpendicular to it have slope $-\frac{3}{2}$.

Why?



$m = +\frac{2}{3}$

"rise" = 2

"run" = 3

"m-perp" is $m_\perp = -\frac{3}{2}$

"rise" = 3

"run" = 2

Example: Find the equation of the line that includes the point $(-1, 2)$ and is parallel to the line with equation $4x - 2y = 6$.

Find slope of given line:

Write in $y = mx + b$ form:

$$4x - 2y = 6$$

$$-2y = -4x + 6$$

$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{6}{-2}$$

$$y = 2x - 3$$

($m = 2$, $b = -3$)

Slope of given line is $m = 2$.

So, the slope of the desired line is also $m = 2$ (because they are parallel)

Put $m = 2$, $x = -1$, $y = 2$ into either $y = mx + b$ or $y - y_1 = m(x - x_1)$

Using point-slope: $y - y_1 = m(x - x_1)$

$$y - 2 = 2(x - (-1))$$

$$y - 2 = 2(x + 1)$$

$$y - 2 = 2x + 2$$

$$+2 \quad +2$$

$$\boxed{y = 2x + 4}$$

Using slope-intercept form:

$y = mx + b$

$m = 2 \Rightarrow$ $y = 2x + b$

$x = -1$, $y = 2$ $\Rightarrow$ $2 = 2(-1) + b$

$$2 = -2 + b$$

$$+2 \quad +2$$

$$4 = b$$

Eqn of line: $\boxed{y = 2x + 4}$

Example: Find the equation of the line that includes the point $(4, 5)$ and is perpendicular to the line with equation $-3x - 5y = 7$.

Find the slope of the given line:

Find slope of given line: $-3x - 5y = 7$

Write in $y = mx + b$ form:

$$-5y = 3x + 7$$

$$\frac{-5y}{-5} = \frac{3x}{-5} + \frac{7}{-5}$$

$$y = -\frac{3}{5}x - \frac{7}{5}$$

($m = -\frac{3}{5}$). So slope of given line is $-\frac{3}{5}$.

So, slope of desired line is $m = +\frac{5}{3}$ (because they are perpendicular)

Using point-slope form: $y - y_1 = m(x - x_1)$

$m = \frac{5}{3}$, $x_1 = 4$, $y_1 = 5 \Rightarrow$ $y - 5 = \frac{5}{3}(x - 4)$

$$y - 5 = \frac{5}{3}x + \frac{5}{3}(-4)$$

$$y - 5 = \frac{5}{3}x - \frac{20}{3}$$

$$y = \frac{5}{3}x - \frac{20}{3} + \frac{5}{1}\left(\frac{3}{3}\right)$$

$$\boxed{y = \frac{5}{3}x - \frac{5}{3}}$$

Previous example:

Work it by using $y = mx + b$:

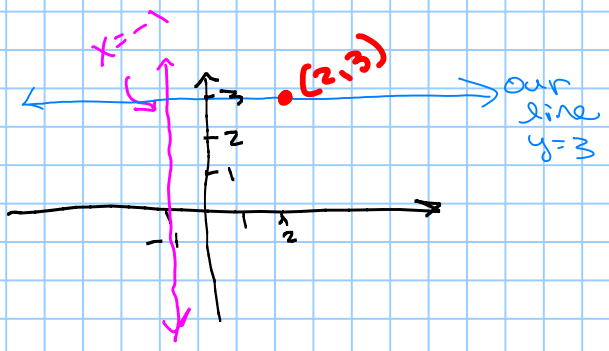
Start with $m = +\frac{5}{3}$.

$x = 4,\ y = 5 \Rightarrow$

$$y = mx + b$$
$$y = \frac{5}{3}x + b$$
$$5 = \frac{5}{3}(4) + b$$
$$5 = \frac{20}{3} + b$$
$$5 - \frac{20}{3} = b$$
$$\left(\frac{3}{3}\right)\frac{5}{1} - \frac{20}{3} = b$$
$$\frac{15}{3} - \frac{20}{3} = b$$
$$-\frac{5}{3} = b$$

So, equation of desired line is $\boxed{y = \frac{5}{3}x - \frac{5}{3}.}$

Ex: Find the equation of the line that includes the point $(2,3)$ and is perpendicular to the line with equation $x = -1$.



The equation of the desired line is $y = 3$.

Example: Find the equation of the line that includes the point $(-2, 4)$ and is (a) parallel and (b) perpendicular to the line with equation $2x - 3y = 1$.

Find slope of given line by writing in $y = mx + b$ form:

$$2x - 3y = 1$$
$$-3y = -2x + 1$$
$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{1}{-3}$$
$$y = \frac{2}{3}x - \frac{1}{3}$$

So slope of given line is $\frac{2}{3}$.

(a) Find the parallel line:

Slope of desired line is also $\frac{2}{3}$ (because it is parallel to the given line)

Using $y = mx + b$:

$y = \frac{2}{3}x + b$ must find b.

$x = -2, y = 4 \Rightarrow 4 = \frac{2}{3}(-2) + b$

$$4 = -\frac{4}{3} + b$$
$$4 + \frac{4}{3} = b$$
$$\left(\frac{3}{3}\right)4 + \frac{4}{3} = b$$
$$\frac{12}{3} + \frac{4}{3} = b$$
$$\frac{16}{3} = b.$$

So equation of desired line is $\boxed{y = \frac{2}{3}x + \frac{16}{3}}$.

(b) Find the perpendicular line:

Slope of given line is $\frac{2}{3}$

So slope of the perpendicular line is $m = -\frac{3}{2}$

Using $y = mx + b$:

$$y = -\frac{3}{2}x + b$$

$x = -2, y = 4 \Rightarrow 4 = -\frac{3}{2}(-2) + b$

$$4 = \frac{6}{2} + b$$
$$4 = 3 + b$$
$$1 = b,$$ so equation of desired line is

$$\boxed{y = -\frac{3}{2}x + 1}$$