

4.1: Multiplication with Exponents

Note Title

2/24/2015

Review of powers/exponents:

Example: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 = \boxed{81}$

Know these powers:

$$2^2 = 4$$

$$2^3 = 8$$

$$2^4 = 16$$

$$2^5 = 32$$

$$2^6 = 64$$

$$2^7 = 128$$

$$2^8 = 256$$

$$2^9 = 512$$

$$2^{10} = 1024$$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 243$$

$$3^6 = 729$$

$$4^2 = 16$$

$$4^3 = 64$$

$$4^4 = 256$$

$$5^2 = 25$$

$$5^3 = 125$$

$$5^4 = 625$$

$$6^2 = 36$$

$$6^3 = 216$$

$$7^2 = 49$$

$$7^3 = 343$$

$$8^2 = 64$$

$$8^3 = 512$$

$$9^2 = 81$$

$$9^3 = 729$$

$$10^2 = 100$$

$$10^3 = 1000$$

$$10^4 = 10000$$

$$10^8 = 100000000$$

Powers of positive and negative numbers:

$$-3^2 = -(3)(3) = \boxed{-9}$$

$$(-3)^2 = (-3)(-3) = \boxed{9}$$

Example: $(-2)^6 = \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} = \boxed{64}$

$$(-2)^5 = \underbrace{(-2)(-2)}_{+} \underbrace{(-2)(-2)}_{+} \underbrace{(-2)}_{-} = \boxed{-32}$$

$$-2^6 = -(2)(2)(2)(2)(2)(2) = \boxed{-64}$$

$$-2^5 = -(2)(2)(2)(2)(2) = \boxed{-32}$$

Important Fact:

* If you raise a negative number to an even power, you get a positive result.

* If you raise a negative number to an odd power, you get a negative result.

Example: $(-3)^4 = \boxed{81}$

Ex: $-(-2)^4 = \boxed{-16}$

Ex: $-(-2)^5 = \underbrace{-(-32)}_{=-1(-32)} = \boxed{32}$

Properties (laws) of Exponents

1) $x^a x^b = x^{a+b}$

Ex: $x^2 x^3 = (xx)(xxx) = x^5$

2) $(x^a)^b = x^{ab}$

Ex: $(x^2)^3 = x^2 x^2 x^2 = (xx)(xx)(xx) = x^6$

3) $\frac{x^a}{x^b} = x^{a-b}$

Ex: $\frac{x^6}{x^2} = \frac{\cancel{xxx}xx}{\cancel{xx}} = \frac{x^4}{1} = x^4$

4) $(xy)^a = x^a y^a$

Ex: $(xy)^3 = (xy)(xy)(xy) = xxx yyy = x^3 y^3$

5) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

Ex: $\left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{xxx}{yyy} = \frac{x^3}{y^3}$

Ex. $\left(\frac{1}{4}\right)^2 = \frac{1^2}{4^2} = \boxed{\frac{1}{16}}$

Ex. $\left(-\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \boxed{\frac{16}{81}}$ [neg raised to even power is positive]

Ex. $\left(-\frac{3}{5}\right)^3 = -\frac{3^3}{5^3} = \boxed{-\frac{27}{125}}$ [neg raised to odd power is negative]

Ex. $\frac{3^{12}}{3^{10}} = 3^{12-10} = 3^2 = \boxed{9}$ ← use property # 3

Ex. $\frac{2^5}{3^4} = \boxed{\frac{32}{81}}$

Ex. $2^2 + 2^3 = 4 + 8 = \boxed{12}$

Ex. $2x^3x^4 = 2x^7$ ← Property #1

Ex. $(6a^4)(-2a) = \boxed{-12a^5}$

← $6a^4(-2a^1) = -12a^{4+1} = -12a^5$ Prop. #1

Ex. $(x^3y^4)^5 = (x^3)^5(y^4)^5$ ← Prop. #4

$= \boxed{x^{15}y^{20}}$ ← Prop. #2

Ex. $\left(\frac{y^3}{x^2}\right)^2 = \frac{(y^3)^2}{x^2}$ ← Prop. #5

$= \boxed{\frac{y^6}{x^2}}$ ← Prop. #2

$$\underline{\text{Ex:}} \quad (-2x^2)^5 = (-2)^5 (x^2)^5 = \boxed{-32x^{10}}$$

$$\underline{\text{Ex:}} \quad (-3x^2)^3 (-2x^4)^4 = (-3)^3 x^6 (-2)^4 x^{16} = -27x^6 (16x^{16}) \\ = \boxed{-432x^{22}}$$

$$\begin{array}{r} 27 \\ \times 16 \\ \hline 162 \\ 270 \\ \hline 432 \end{array}$$

$$\underline{\text{Ex:}} \quad \frac{(2x)^7}{(2x)^4} = (2x)^3 = 2^3 x^3 = \boxed{8x^3}$$

$$\underline{\text{OR}} \quad \frac{(2x)^7}{(2x)^4} = \frac{2^7 x^7}{2^4 x^4} = 2^3 x^3 = \boxed{8x^3}$$

$$\underline{\text{Example:}} \quad \frac{(2x^2yz)(-x^4y^2z^3)^4}{(3xy)^2} = \frac{(2x^2yz)(-1x^4y^2z^3)^4}{(3xy)^2}$$

$$= \frac{(2x^2yz)(-1)^4 x^{16} y^8 z^{12}}{(3)^2 x^2 y^2} = \frac{(2x^2yz)(1) x^{16} y^8 z^{12}}{9 x^2 y^2}$$

$$= \frac{2x^{18} y^9 z^{13}}{9x^2 y^2} = \boxed{\frac{2x^{16} y^7 z^{13}}{9}} = \boxed{\frac{2}{9} x^{16} y^7 z^{13}}$$

4.2: Division with Exponents and Negative Exponents

Skip Scientific Notation problems until the end of 4.2

Definition of a Negative Exponent

$$x^{-n} = \frac{1}{x^n}$$

$$\underline{\text{Note:}} \quad x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$$

$$\underline{\text{Also:}} \quad \frac{1}{x^{-n}} = \frac{x^n}{1} = x^n$$

$$x^{-1} = \frac{1}{x} \quad \text{reciprocal!}$$

$$\underline{\text{Ex:}} \quad 3^{-2} = \frac{3^{-2}}{1} = \frac{1}{3^2} = \boxed{\frac{1}{9}}$$

In all examples in this section, simplify and write answers without negative exponents.

$$\underline{\text{Ex:}} \quad -2^{-4} = \frac{-2^{-4}}{1} = -\frac{2^{-4}}{1} = -\frac{1}{2^4} = \boxed{-\frac{1}{16}}$$

$$\underline{\text{Ex:}} \quad (-3)^{-4} = \frac{(-3)^{-4}}{1} = \frac{1}{(-3)^4} = \boxed{\frac{1}{81}}$$

$$\underline{\text{Ex:}} \quad \left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \boxed{\frac{9}{4}}$$

$$\underline{\text{or}} \quad \left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \boxed{\frac{9}{4}} \quad \leftarrow \text{same}$$