4.3: Operations with Monomials

Note Title 3/3/2015

Polynomial: expression composed of terms added together, in which each term has variables to positive exponents only.

Examples of polynomials:

$-2x^2 + 5x^3 - 8$

$3x + 1$

$\frac{2}{3}x^4 - \frac{7}{2}x^8 + \frac{1}{8}x$

$x^3y^2z + 4x^2y^2 - 7y^4$

Not polynomials: $2x^{-5} - 4x^{-3}$

$\frac{1}{x^4} + \frac{5}{x^3}$ ← same as $x^{-4} + 5x^{-3}$

$\sqrt{x}$ ← same as $x^{1/2}$

$\sqrt{x^2 + 5x}$

Degree of a polynomial:

* If it has 1 variable only, then the degree is the largest exponent
* If it has 2 or more variables, then the degree is the largest total exponent in a term

Example: the degree of $-8x^3 - 2x^5 + 3x^2 - 9$ is 5.

Example: The degree of $4x - x^7$ is 7.
(it's a 7th degree polynomial)

Leading term: Term with the highest exponent.

Leading coefficient: Coefficient of the leading term.

Ex: $-8x^3 - 2x^5 + 3x^2 - 9$

Leading term: $-2x^5$ (Degree is 5)

Leading Coefficient: -2

Could write this polynomial as $-2x^5 - 8x^3 + 3x^2 - 9$

Ex: $2x - x^8$

Leading Term: $-x^8$

Leading Coefficient: -1

Degree: 8

Ex: What is the degree of this polynomial?

$$\underbrace{2x^2y^3}_{\substack{\text{total}\\\text{exponent}\\2+3=5}} + \underbrace{5x^3y^4}_{\substack{\text{total}\\\text{exponent}\\3+4=7}} - \underbrace{7xy^5}_{\substack{\text{total}\\\text{exponent}\\1+5=6}} - \underbrace{8y^4}_{\substack{\text{total exponent}\\=4}}$$

Biggest

$\boxed{\text{So, the degree is 7.}}$

Special terminology for certain polynomials:

Polynomial with 3 terms: Trinomial

Polynomial with 2 terms: Binomial

Polynomial with 1 term: Monomial

Ex: $3-x^4$ is a 4^{th} degree binomial.

$4x^2-8x+13$ is a 2^{nd} degree trinomial.

$2x^8y^2$ is a 10^{th} degree monomial.

Multiplying monomials:

Ex: $(-2x^4)(3x^6) = \boxed{-6x^{10}}$

Ex: $(-8x^2y^4)(-2x^3yz^5)$

$= \boxed{16x^5y^5z^5}$

Dividing Monomials

Ex: $\dfrac{2\cancel{4}^{8}x^6}{-\cancel{3}_{1}x^2} = -\dfrac{8x^4}{1} = \boxed{-8x^4}$

Ex: $\dfrac{2x^3}{8x^9} = \boxed{\dfrac{1}{4x^6}}$ Could write this as $\frac{1}{4}x^{-6}$

(not equal to $4x^6$)!!!

4.4: Addition and Subtraction of Polynomials

simplify

Example: $(3x^2 - 7x + 10) + (-2x^2 - 8x - 5)$

$= 3x^2 - 7x + 10 - 2x^2 - 8x - 5$

$= \boxed{x^2 - 15x + 5}$

Example: $(2x^3 - 5x^2 - 3x + 6) - (-x^2 + 5x - 8)$

$= 2x^3 - 5x^2 - 3x + 6 - 1(-x^2 + 5x - 8)$

$= 2x^3 - 5x^2 - 3x + 6 + x^2 - 5x + 8$

$= \boxed{2x^3 - 4x^2 - 8x + 14}$

Ex: 4.4 #32) $\left(4x^3 - \frac{2}{5}x^2 + \frac{3}{8}x - 1\right) - \left(\frac{9}{2}x^3 + \frac{1}{4}x^2 - x + \frac{5}{6}\right)$

$= 4x^3 - \frac{2}{5}x^2 + \frac{3}{8}x - 1 - \frac{9}{2}x^3 - \frac{1}{4}x^2 + x - \frac{5}{6}$

$= 4x^3 - \frac{9}{2}x^3 - \frac{2}{5}x^2 - \frac{1}{4}x^2 + \frac{3}{8}x + x - 1 - \frac{5}{6}$

$= \frac{4x^3}{1}\left(\frac{2}{2}\right) - \frac{9x^3}{2} - \frac{2}{5}\left(\frac{4}{4}\right)x^2 - \frac{1}{4}\left(\frac{5}{5}\right)x^2 + \frac{3}{8}x + \frac{1}{1}x\left(\frac{8}{8}\right) - \frac{1}{1}\left(\frac{6}{6}\right) - \frac{5}{6}$

$= \frac{8}{2}x^3 - \frac{9}{2}x^3 - \frac{8}{20}x^2 - \frac{5}{20}x^2 + \frac{3}{8}x + \frac{8}{8}x - \frac{6}{6} - \frac{5}{6}$

$= \boxed{-\frac{1}{2}x^3 - \frac{13}{20}x^2 + \frac{11}{8}x - \frac{11}{6}}$

4.4 #27 $\left(\frac{2}{3}x^2 - \frac{1}{5}x - \frac{3}{4}\right) + \left(\frac{4}{3}x^2 - \frac{4}{5}x + \frac{7}{4}\right)$

$$= \frac{2}{3}x^2 - \frac{1}{5}x - \frac{3}{4} + \frac{4}{3}x^2 - \frac{4}{5}x + \frac{7}{4}$$

$$= \frac{2}{3}x^2 + \frac{4}{3}x^2 - \frac{1}{5}x - \frac{4}{5}x - \frac{3}{4} + \frac{7}{4}$$

$$= \frac{6}{3}x^2 - \frac{5}{5}x + \frac{4}{4}$$

$$= 2x^2 - 1x + 1$$

$$= \boxed{2x^2 - x + 1}$$

4.5: Multiplication of Polynomials

Multiplying polynomials when one is a monomial

Example: $3x^3(2x^2 - 8x + 7)$

$$= 3x^3(2x^2) + 3x^3(-8x) + 3x^3(7)$$

$$= \boxed{6x^5 - 24x^4 + 21x^3}$$

Example: $-5x^2y^3(-xy^4 + 8xy - 3x^3y^5 - 2y^2)$

$$= -5x^2y^3(-xy^4) - 5x^2y^3(8xy) - 5x^2y^3(-3x^3y^5) - 5x^2y^3(-2y^2)$$

$$= \boxed{5x^3y^7 - 40x^3y^4 + 15x^5y^8 + 10x^2y^5}$$

Ex: $\frac{3}{4}x^2\left(\frac{1}{2}x^3 - \frac{2}{5}x^2 - \frac{1}{8}x + \frac{3}{4}\right)$

$= \frac{3}{4}x^2\left(\frac{1}{2}x^3\right) + \frac{3}{4}x^2\left(-\frac{2}{5}x^2\right) + \frac{3}{4}x^2\left(-\frac{1}{8}x\right) + \frac{3}{4}x^2\left(\frac{3}{4}\right)$

$= \frac{3}{8}x^5 - \frac{\cancel{6}^{3}}{\cancel{20}_{10}}x^4 - \frac{3}{32}x^3 + \frac{9}{16}x^2$

$= \boxed{\frac{3}{8}x^5 - \frac{3}{10}x^4 - \frac{3}{32}x^3 + \frac{9}{16}x^2}$

Multiplying 2 polynomials that have 2 or more terms

Recall: Distributive property

$c(a+b) = ca + cb$

$(a+b)(c) = ca + cb = ac + bc$

Example: $(\underbrace{3x}_{a} + \underbrace{5}_{b})\underbrace{(x-8)}_{c}$

$= \underbrace{3x}_{a}\underbrace{(x-8)}_{c} + \underbrace{5}_{b}\underbrace{(x-8)}_{c}$

$= 3x^2 - 24x + 5x - 40$

$= \boxed{3x^2 - 19x - 40}$

Ex: $(2x-1)(6x-5)$

$= 2x(6x-5) - 1(6x-5)$

$= 12x^2 - 10x - 6x + 5$

$= \boxed{12x^2 - 16x + 5}$

Ex: $(-2x+8)(x^2-9x+3)$

$= -2x(x^2-9x+3) + 8(x^2-9x+3)$

$= -2x^3 + 18x^2 - 6x + 8x^2 - 72x + 24$

$= \boxed{-2x^3 + 26x^2 - 78x + 24}$

Ex: $(x^2-4x-3)(5x^2+7x-8)$

$= x^2(5x^2+7x-8) - 4x(5x^2+7x-8) - 3(5x^2+7x-8)$

$$= 5x^4 + 7x^3 - 8x^2 - 20x^3 - 28x^2 + 32x - 15x^2 - 21x + 24$$

$= \boxed{5x^4 - 13x^3 - 51x^2 + 11x + 24}$

Plan for next time: 15 min on 4.6
15 min on 4.7
Rest of 4.8

Prereading Assignment:

Read 4.6 (pp 358-360) and 4.7 (pp 365-366)

Rework Example 5 on p. 366
similar problem: Matched problem #5