# 4.6: Special Products

Note Title 3/5/2015

**Recall:** multiplying polynomials

$$(x^2-3)(2x^2-6x+5)$$

$$= x^2(2x^2-6x+5) - 3(2x^2-6x+5)$$

$$= 2x^4 - 6x^3 + \underline{5x^2} - \underline{6x^2} + 18x - 15$$

$$= \boxed{2x^4 - 6x^3 - x^2 + 18x - 15}$$

**Ex:** $(-3x^2-8x+1)(4x^2+9x-3)$

$$= -3x^2(4x^2+9x-3) - 8x(4x^2+9x-3) + 1(4x^2+9x-3)$$

$$\begin{aligned} = -12x^4 - 27x^3 &+ 9x^2 \\ -32x^3 &- 72x^2 + 24x \\ &+ 4x^2 + 9x - 3 \end{aligned}$$

$$= \boxed{-12x^4 - 59x^3 - 59x^2 + 33x - 3}$$

## The "FOIL" Method (for multiplying 2 binomials)

F: First

O: Outer

I: Inner

L: Last

**Ex:** $(2x+5)(3x-8)$

First Last Inner Outer

$$= 6x^2 - 16x + 15x - 40$$

$$= \boxed{6x^2 - x - 40}$$

Using distributive property:

$$2x(3x-8) + 5(3x-8)$$

$$= 6x^2 - 16x + 15x - 40 = \boxed{6x^2 - x - 40}$$

Ex: $(5x-2)(3y-7)$

$= \boxed{15xy - 35x - 6y + 14}$ (no like terms)

## Perfect Squares

Ex: $(x-6)^2$. Simplify.

$(x-6)^2$

$= (x-6)(x-6)$

$= x^2 - 6x - 6x + 36$

$= \boxed{x^2 - 12x + 36}$

**Perfect Square Pattern**

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

Check:

$(a+b)^2$

$= (a+b)(a+b)$

$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$ ✓

Ex: $(x+4)^2$

$= \boxed{x^2 + 8x + 16}$

Ex: $(2x-5)^2$

$= \boxed{4x^2 - 20x + 25}$

write it out:

$(2x-5)(2x-5)$

$= 4x^2 - 10x - 10x + 25$

$= \boxed{4x^2 - 20x + 25}$

Example: $(x-4)(x+4)$

$x^2 + 4x - 4x - 16$

$\boxed{x^2 - 16}$

**Difference of 2 Squares Pattern**

$(a+b)(a-b) = a^2 - b^2$

Check it: $(a+b)(a-b) = a^2 - \cancel{ab} + \cancel{ab} - b^2$

$= a^2 - b^2$

Ex: $(x-7)(x+7)$

$= \boxed{x^2 - 49}$

Note: $(x-7)(x+7)$

$= x^2 + 7x - 7x - 49$

$= \boxed{x^2 - 49}$

Ex: $(3x+5)(3x-5)$

$= (3x)^2 - (5)^2$

$= \boxed{9x^2 - 25}$

Note: $(3x+5)(3x-5)$

$= 9x^2 - \cancel{15x} + \cancel{15x} - 25$

$= \boxed{9x^2 - 25}$

# 4.7: Dividing a polynomial by a monomial

**Ex:** Divide.

$$\frac{8x^3 - 2x^2}{2x}$$

$$\frac{\cancel{8}^{4}x^3}{\cancel{2}_{1}x} - \frac{\cancel{2}^{1}x^2}{\cancel{2}_{1}x}$$

$$= \frac{4x^2}{1} - \frac{x}{1}$$

$$= \boxed{4x^2 - x}$$

**Note:**

$$\frac{2}{7} + \frac{3}{7}$$

$$= \frac{2+3}{7}$$

$$= \frac{5}{7}$$

**Ex:** Divide.

$$\frac{9x^4 - 12x^3 + 8x^2 + 21}{3x^2}$$

split into separate fractions

$$\frac{9x^4 - 12x^3 + 8x^2 + 21}{3x^2} = \frac{\cancel{9}^{3}x^4}{\cancel{3}_{1}x^2} - \frac{\cancel{12}^{4}x^3}{\cancel{3}_{1}x^2} + \frac{8\cancel{x^2}^{1}}{3\cancel{x^2}_{1}} + \frac{\cancel{21}^{7}}{\cancel{3}_{1}x^2}$$

$$= \frac{3x^2}{1} - \frac{4x}{1} + \frac{8}{3} + \frac{7}{x^2}$$

$$= \boxed{3x^2 - 4x + \frac{8}{3} + \frac{7}{x^2}}$$

**Ex:** $$\frac{2x^4y^3 + 8x^3y^3 - 4y^2}{-2xy^2}$$

$$= \frac{\cancel{2}^{1}x^4y^3}{-\cancel{2}_{1}xy^2} + \frac{\cancel{8}^{4}x^3y^3}{-\cancel{2}_{1}xy^2} - \frac{\cancel{4}^{2}y^2}{-\cancel{2}_{1}xy^2}$$

$$= -\frac{1x^3y}{1} - \frac{4x^2y}{1} + \frac{2}{1x} = \boxed{-x^3y - 4x^2y + \frac{2}{x}}$$

# 4.8: Dividing a Polynomial by a Polynomial

(when divisor has 2 or more terms)

## Polynomial Long Division

Example: Divide $\frac{379}{12}$.

$$\begin{array}{r} 31\phantom{)} \\ 12\,\overline{)\,379} \\ \underline{36\phantom{9}} \\ 19 \\ \underline{12} \\ 7 \end{array}$$

Write answer: $\frac{379}{12} = 31 + \frac{7}{12}$ (Usually written $31\frac{7}{12}$)

Check your work: $12(31) + 7$
$= 372 + 7 = 379$ ✓

$$\begin{array}{r} 31 \\ \times\, 12 \\ \hline 62 \\ 31\phantom{0} \\ \hline 372 \end{array}$$

Terminology Review:

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\,\overline{)\,\text{Dividend}} \\ \hline \text{Remainder} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

Ex: Divide.

$$\frac{3x^2+7x+9}{x+2}$$

Note: Could write as

$$\frac{3x^2}{x+2} + \frac{7x}{x+2} + \frac{9}{x+2}$$

but they don't reduce.

We need to do polynomial long division.

$\frac{3x^2}{x}$ $\quad$ $\frac{1x}{x}$

$$\begin{array}{r|llll}
 & 3x & + 1 & & \\ \hline
x+2 & 3x^2 & + 7x & + 9 & \\
 & -(3x^2 & + 6x) & & \leftarrow 3x(x+2) = 3x^2+6x \\ \hline
 & 0 & + 1x & + 9 & \\
 & & -(x & + 2) & \leftarrow 1(x+2) = x+2 \\ \hline
 & & & 7 &
\end{array}$$

Write answer: $\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$ ← answer

Check: (Quotient)(Divisor) + Remainder = Dividend

$(3x+1)(x+2)+7$

$= 3x^2+6x+1x+2+7 = 3x^2+7x+9$ ✓

Note: If you let $x=10$, this turns into $\frac{379}{12} = 31+\frac{7}{12}$

Ex: Divide. $\dfrac{x^3 - 6x^2 + x + 14}{x - 5}$

$\dfrac{x^3}{x}$ $\dfrac{-1x^2}{x}$ $\dfrac{-4x}{x}$

$$
\begin{array}{r|l}
 & x^2 - x - 4 \\
\hline
x - 5 & x^3 - 6x^2 + x + 14 \\
 & -(x^3 - 5x^2) \\
\hline
 & -1x^2 + x \\
 & -(-x^2 + 5x) \\
\hline
 & -4x + 14 \\
 & -(-4x + 20) \\
\hline
 & -6
\end{array}
$$

$x^2(x-5) = x^3 - 5x^2$

$-x(x-5) = -x^2 + 5x$

$-4(x-5) = -4x + 20$

Write answer:

$$\frac{x^3 - 6x^2 + x + 14}{x - 5} = \boxed{x^2 - x - 4 + \frac{-6}{x-5}} = \boxed{x^2 - x - 4 - \frac{6}{x-5}}$$

usually written this way.

(either way is ok)

Check answer:

$(x-5)(x^2 - x - 4) - 6$

$= x^3 - x^2 - 4x$
$\quad - 5x^2 + 5x + 20 - 6$

$= x^3 - 6x^2 + x + 14$ ✓

**Important:** When doing polynomial long division, insert 0's as placeholders for missing powers of $x$ ✱ ✱

Example of this:

To divide $\dfrac{x^3 + 5x}{2x - 7}$, write:

$$2x - 7 \overline{\big)\, x^3 + 0x^2 + 5x + 0}$$