4.8: Long Division of Polynomials (cont'd)

Note Title 3/17/2015

Example: Divide. $\dfrac{8x^3 - 10x^2 + x - 2}{2x - 1}$

$\frac{8x^3}{2x}$ $\frac{-6x^2}{2x}$ $\frac{-2x}{2x}$

$$2x-1 \;\overline{)\; 8x^3 - 10x^2 + x - 2}$$

$4x^2 - 3x - 1$

$\pm(8x^3 \mp 4x^2)$

$-6x^2 + x$

$\pm(-6x^2 \pm 3x)$

$-2x - 2$

$\pm(-2x \pm 1)$

-3

Write our answer: $\dfrac{8x^3 - 10x^2 + x - 2}{2x - 1} = \boxed{4x^2 - 3x - 1 + \dfrac{-3}{2x-1}}$

usually written $\boxed{4x^2 - 3x - 1 - \dfrac{3}{2x-1}}$

Check your answer: $(2x-1)(4x^2 - 3x - 1) - 3$

$= 8x^3 - 6x^2 - 2x$
$\quad -4x^2 + 3x + 1 - 3$

$= 8x^3 - 10x^2 + x - 2$ (equal to dividend/numerator)

✓ OK

Example: Divide. $\dfrac{3x^4 - 8x^2 + 2x - 7}{2x^3}$

Try long division: $2x^3 \overline{)\,3x^4 + 0x^3 - 8x^2 + 2x - 7}$

When dividing by a monomial (1 term), break into separate fractions:

$$\frac{3x^4 - 8x^2 + 2x - 7}{2x^3} = \frac{3x^4}{2x^3} - \frac{8x^2}{2x^3} + \frac{2x}{2x^3} - \frac{7}{2x^3}$$

$$= \frac{3x}{2} - \frac{4}{x} + \frac{1}{x^2} - \frac{7}{2x^3}$$

Important: Arrange terms in order, starting with the largest exponent.

★ Insert "placeholders" (example: $0x^3$, $0x^2$) for missing powers of x

Example: Divide. $\dfrac{3x^4 - 50x^2 - x}{x - 4}$

$\frac{3x^4}{x}$ $\frac{12x^3}{x}$ $\frac{-2x^2}{x}$ $\frac{-9x}{x}$

$$3x^3 + 12x^2 - 2x - 9$$

$$x-4 \overline{)\, 3x^4 + 0x^3 - 50x^2 - x + 0}$$

$\ominus\ (-3x^4 \pm 12x^3)$

$$12x^3 - 50x^2$$

$\ominus\ (12x^3 \pm 48x^2)$

$$-2x^2 - x$$

$\pm\ (\pm 2x^2 \mp 8x)$

$$-9x + 0$$

$\pm\ (\pm 9x \mp 36)$

$$-36$$

$$\frac{3x^4 - 50x^2 - x}{x-4} = \boxed{3x^3 + 12x^2 - 2x - 9 + \frac{-36}{x-4}}$$

Check answer: $(x-4)(3x^3 + 12x^2 - 2x - 9) - 36$

$$= 3x^4 + 12x^3 - 2x^2 - 9x$$
$$\quad - 12x^3 - 48x^2 + 8x + 36 - 36$$

$$= 3x^3 - 50x^2 - x \quad \checkmark \text{ok}$$

Example: Divide. $\dfrac{10x^3 + 21x^2 - 5}{2x^2 + 5x + 2}$

$$
\begin{array}{r}
5x - 2 \\
2x^2 + 5x + 2 \overline{)\,10x^3 + 21x^2 + 0x - 5} \\
\underline{\pm(\mp 10x^3 \mp 25x^2 \mp 10x)} \\
-4x^2 - 10x - 5 \\
\underline{\pm(\mp 4x^2 \mp 10x \mp 4)} \\
-1
\end{array}
$$

$$\frac{10x^3 + 21x^2 - 5}{2x^2 + 5x + 2} = \boxed{5x - 2 + \frac{-1}{2x^2 + 5x + 2}}$$

Check: $(2x^2 + 5x + 2)(5x - 2) - 1$

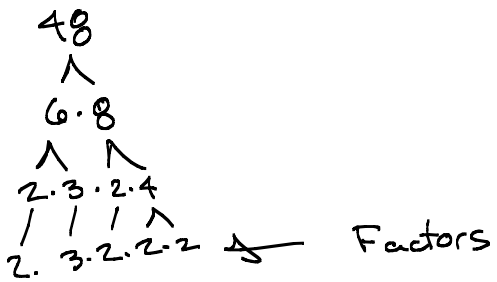
$= 10x^3 - 4x^2$
$\quad + 25x^2 - 10x$
$\quad\quad + 10x - 4 - 1$

$= 10x^3 + 21x^2 - 5$ ✓ ok

Chapter 5: Factoring Polynomials

Factoring: breaking down into factors

Factoring a number:



Polynomials

$(x+3)(x+4)$ ← factored form

$x^2+7x+12$ ← unfactored form

5.1: The Greatest Common Factor and Factoring by Grouping

To find the Greatest Common Factor (GCF)

* Find the largest number that divides into all the coefficients
* Find the highest power of each variable that is a factor of all the terms.
  (if a variable shows up in all the terms, choose the smallest power)
* Multiply the numerical part and the variable part together to get the GCF.

Example: Factor out the GCF.

$15x^4 - 6x^2$ GCF: $3x^2$

$3x^2\left(\frac{15x^4}{3x^2} - \frac{6x^2}{3x^2}\right)$

$= 3x^2(5x^2 - 2)$

Check: $3x^2(5x^2-2) = 15x^4 - 6x^2$ OK

should match original

Ex: Factor.

$48x^3 - 32x$

GCF: $8x$

$= 8x(6x^2 - 4)$

$= 8x(2)(3x^2 - 2)$

$= \boxed{16x(3x^2 - 2)}$

Check: $8x(6x^2 - 4) = 48x^3 - 32x$ ✓

Check: $16x(3x^2 - 2)$

$= 48x^3 - 32x$ ✓

(We did not get the greatest common factor)

If we would have factored out the GCF from the beginning:

$48x^3 - 32x$

GCF: $16x$

$= \boxed{16x(3x^2 - 2)}$ ✓

Ex: $32x^3 - 24x + 6$

GCF: 2

$= \boxed{2(16x^3 - 12x + 3)}$

Ex: Factor. $-30x^4y^3z + 12x^3y^6 - 42x^4y^2z$

GCF: $6x^3y^2$

$= \boxed{6x^3y^2(-5xyz + 2y^4 - 7xz)}$

Check: $6x^3y^2(-5xyz + 2y^4 - 7xz)$

$= -30x^4y^3z + 12x^3y^6 - 42x^4y^2z$ ✓

Work on #1-35 on Section 5.1