5.1: Greatest Common Factor and Factoring by Grouping (cont'd)

Note Title 3/24/2015

Example: Factor.

$$12x^3 - 8x^2 + 32x$$

GCF: $4x$

$$= \boxed{4x(3x^2 - 2x + 8)}$$

Check: $4x(3x^2 - 2x + 8)$

$= 12x^3 - 8x^2 + 32x$ ✓

Ex: $3x^3y^4 - 5x^2y^2 + x^2y$

GCF: x^2y

$$x^2y(3xy^3 - 5y + 1)$$

Factoring by Grouping

Example: Factor.

$$3x^3 - 7x^2 + 15x - 35$$

$$= (3x^3 - 7x^2) + (15x - 35)$$

$$= x^2(3x - 7) + 5(3x - 7)$$

(1st term, 2nd term)

$$= \boxed{(3x-7)(x^2+5)}$$

Check: $(3x-7)(x^2+5)$

$= 3x^3 + 15x - 7x^2 - 35$

Ex: Factor.

$4x^3 - 3x^2 - 36x + 27$

$= (4x^3 - 3x^2) + (-36x + 27)$

$= x^2(4x-3) - 9(4x-3)$

$= (4x-3)(x^2-9)$

$= \boxed{(4x-3)(x+3)(x-3)}$

Check:

$(4x-3)(x^2-9)$

$= 4x^3 - 36x - 3x^2 + 27$ ✓ OK

Note: $(x+3)(x-3)$

$= x^2 - 3x + 3x - 9$

$= x^2 - 9$

we haven't learned yet about how to factor $x^2 - 9$

Ex: $2a^2b - 16a^2 + b - 8$ Factor.

$= (2a^2b - 16a^2) + (b - 8)$

$= 2a^2(b-8) + 1(b-8)$

$= \boxed{(b-8)(2a^2+1)}$

also correct: $\boxed{(2a^2+1)(b-8)}$

Check: $(2a^2+1)(b-8)$

$= 2a^2b - 16a^2 + b - 8$ ✓

Ex: $-16x^2y - 4x^2 + 24xy + 6x$

GCF: $2x$

$= 2x(-8xy - 2x + 12y + 3)$

$= 2x[(-8xy - 2x) + (12y + 3)]$

$= 2x[-2x(4y+1) + 3(4y+1)]$

$= 2x[(4y+1)(-2x+3)]$

$= \boxed{2x(4y+1)(-2x+3)}$

Check: $2x(4y+1)(-2x+3)$

$= 2x(-8xy + 12y - 2x + 3)$

$= -16x^2y + 24xy - 4x^2 + 6x$

Ex: $7xy^2 - 20x + 2y^2 - 70x^2$

$= (7xy^2 - 20x) + (2y^2 - 70x^2)$

$= x(7y^2 - 20) + 2(y^2 - 35x^2)$

Parentheses don't match!

Switch 2 terms.

(I'll switch 2nd & 3rd terms)

$7xy^2 + 2y^2 - 20x - 70x^2$

$(7xy^2 + 2y^2) + (-20x - 70x^2)$

$y^2(7x+2) - 10x(2 + 7x)$

$= \boxed{(7x+2)(y^2 - 10x)}$

Check ✓: $(7x+2)(y^2 - 10x)$

$= 7xy^2 - 70x^2 + 2y^2 - 20x$ ✓

5.2: Factoring Trinomials (when the leading coefficient is 1)

For factoring trinomials, write all polynomials in order of descending powers (start with biggest power and work your way down)

Ex: Factor.

(+) signs are the same (either both + or both −)

$x^2 + 6x + 8$

$\boxed{(x+2)(x+4)}$ ← final answer

8

1 · 8

2 · 4

Check: $(x+2)(x+4)$

$= x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$ ✓ ok

Ex: $x^2 + 13x - 30$

(−) signs are opposite (one is +, one is −)
want a difference of 13

$(x - 2)(x + 15)$

30
1 · 30
2 · 15
3 · 10
5 · 6

Check: $(x-2)(x+15)$
$= x^2 + 15x - 2x - 30$
$= x^2 + 13x - 30$ ✓ok

Ex: $x^2 + 13x + 30$

(+) same signs want a sum of 13

$(x + 3)(x + 10)$

30
1 · 30
2 · 15
3 · 10
5 · 6

Check: $(x+3)(x+10)$
$= x^2 + 10x + 3x + 30$
$= x^2 + 13x + 30$ ✓ok

Ex: $x^2 - 13x - 30$

(−) signs are opposite want a difference of 13

$(x + 2)(x - 15)$

30
1 · 30
2 · 15
3 · 10
5 · 6

Check: $(x+2)(x-15)$
$= x^2 - 15x + 2x - 30 = x^2 - 13x - 30$ ✓

Ex: $x^2 - 13x + 30$

(+) signs are the same want a sum of 13

$(x - 3)(x - 10)$

30
1 · 30
2 · 15
3 · 10
5 · 6

Check: $(x-3)(x-10)$
$= x^2 - 10x - 3x + 30$
$= x^2 - 13x + 30$ ✓

Ex: $x^2 - 10x - 24$

(−) signs are opposite difference of 10

$(x + 2)(x - 12)$

24
1 · 24
2 · 12
3 · 8
4 · 6

Check: $x^2 - 12x + 2x - 24$
$x^2 - 10x - 24$ ✓

Ex: $x^2 + 6x + 9$ (+) same signs, sum of 6

$(x+3)(x+3)$

Check: $x^2+3x+3x+9$

x^2+6x+9 ✓

$\boxed{(x+3)^2}$ ← final answer

9
1·9
3·3

Ex: $x^2 + 7x - 12$ (−) signs are opposite, want a difference of 7

This polynomial is not factorable.

This polynomial is prime.

not possible!

12
1·12
2·6
3·4

Prime number: a natural number that can only be factored as a product of 1 and itself.

Ex: 13 is prime
$= 1 \cdot 13$

Ex: 8 is composite. (not prime)
$8 = 2 \cdot 4$

A prime polynomial: cannot be factored (using integer coefficients) except by factoring out 1 or −1.

Ex: $x^2 - 11xy - 12y^2$ (−) signs are opposite, want a difference of 11

$(x+1y)(x-12y)$

Check: $x^2 - 12xy + 1xy - 12y^2$

$x^2 - 11xy - 12y^2$ ✓

Answer: $\boxed{(x+y)(x-12y)}$ or $\boxed{(x-12y)(x+y)}$

12
1·12
2·6
3·4

Ex: $-x^2+14x-40$

$-1(x^2-14x+40)$ (+) same signs, sum of 14

$-(x-4)(x-10)$

40
1 · 40
2 · 20
4 · 10
5 · 8

check: $(x-4)(x-10)$

$= x^2-10x-4x+40$ ✓

Thursday: Come ready for a quiz over polynomial long division.