5.3: Factoring Trinomials (when the leading coefficient is greater than 1)

Note Title 3/26/2015

Ex: Factor.

$x^2 + x - 12$ (−) signs are opposite; want a difference of 1

$(x-3)(x+4)$

check $(x-3)(x+4)$
$= x^2 + 4x - 3x - 12$
$= x^2 + x - 12$ ✓ ok

Also correct: $(x+4)(x-3)$

12
1 · 12
2 · 6
3 · 4

Try $(x+3)(x-4)$
check: $x^2 - 4x + 3x - 12$
$= x^2 - x - 12$
↑ wrong sign on middle term

Ex: Factor.

$2x^2 - 30x + 108$
$= 2(x^2 - 15x + 54)$ (+) signs are the same; want a sum of 15
$= 2(x-6)(x-9)$

check: $(x-6)(x-9)$
$= x^2 - 9x - 6x + 54$
$= x^2 - 15x + 54$ ✓

54
1 · 54
2 · 27
3 · 18
6 · 9

Ex: $x^2 - 3x - 180$ (−) signs are opposite; want a difference of 3

$(x+12)(x-15)$

Check: $x^2 - 15x + 12x - 180$
$= x^2 - 3x - 180$ ✓

180
18 · 10
3 · 6 · 2 · 5
3 · 2 · 3 · 2 · 5

1
12
15
60
12
180

6 · 30
9 · 20
12 · 15
36 · 5
45 · 4

1 · 180
2 · 90
3 · 60
18 · 10

Ex.: Factor.

$2x^2 + 9x + 10$ (+) same signs want a sum of $9x$ for middle term

Try $(2x + 1)(x + 10)$

$2x^2$ → $2x \cdot x$; 10 → $1 \cdot 10$, $2 \cdot 5$

Check: $2x^2 + 20x + 1x + 10$
$= 2x^2 + 21x + 10$ No!

Try $(2x + 10)(x + 1)$

Check: $2x^2 + 2x + 10x + 10$
$= 2x^2 + 12x + 10$ No!

Try $(2x + 2)(x + 5)$

check: $2x^2 + 10x + 2x + 10$
$= 2x^2 + 12x + 10$ No!

Try $\boxed{(2x + 5)(x + 2)}$

Check: $2x^2 + 4x + 5x + 10$
$= 2x^2 + 9x + 10$ ✓ok

Ex: Factor.

$2x^2 - 25x + 12$ (+) same signs want sum of $25x$ for my middle ter

$\boxed{(2x - 1)(x - 12)}$

$2x^2$ → $2x \cdot x$; 12 → $1 \cdot 12$, $2 \cdot 6$, $3 \cdot 4$; $24x$, $1x$

Check: $2x^2 - 24x - 1x + 12$
$= 2x^2 - 25x + 12$ ✓ok

Ex: Factor.

$4x^2 - 7x - 15$ (−) signs are opposite want a difference of $7x$ for my middle term

$\boxed{(4x + 5)(x - 3)}$

$4x^2$ → $2x \cdot 2x$, $4x \cdot x$; 15 → $1 \cdot 15$, $3 \cdot 5$; $5x$, $12x$

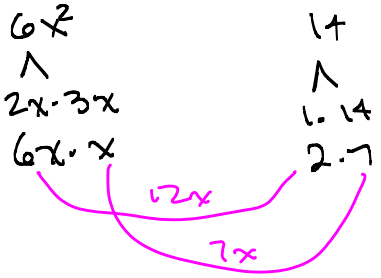
Check: $4x^2 - 12x + 5x - 15$
$= 4x^2 - 7x - 15$ ✓ok

Ex: Factor

$6x^2 + 19x + 14$

(+) same signs sum of 19x for middle term

$(6x + 7)(x + 2)$



Check: $6x^2 + 12x + 7x + 14$

$= 6x^2 + 19x + 14$ ✓ ok

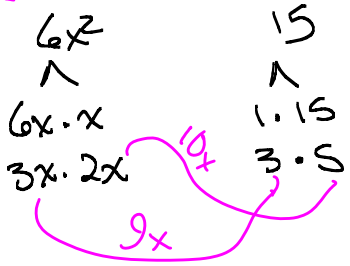
Ex: Factor.

$-6x^2 - x + 15$

$= -(6x^2 + x - 15)$

(-) signs are opposite want a difference of 1x for middle term

$= -(3x + 5)(2x - 3)$



Check: $(3x+5)(2x-3)$

$= 6x^2 - 9x + 10x - 15$

$= 6x^2 + x - 15$ ✓

5.4: Difference of 2 squares and perfect square trinomials

Difference of 2 squares Factorization

$$a^2 - b^2 = (a+b)(a-b)$$

Check: $(a+b)(a-b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$ ✓

Ex: Factor.

$x^2 - 25$

$= x^2 - 5^2$

$= (x+5)(x-5)$

Check: $x^2 - 5x + 5x - 25$

$= x^2 - 25$

Ex: $4x^2 - 9y^2$

$= (2x)^2 - (3y)^2$

$= \boxed{(2x+3y)(2x-3y)}$ Check it!

Also correct: $\boxed{(2x-3y)(2x+3y)}$

Important Fact:

$x^2 +$ positive is always prime. (can't be factored)

Ex: Factor x^2+36.

Prime

Perfect Square Trinomials:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

Ex: Factor $x^2 - 6x + 9$

$(x-3)(x-3)$

$= \boxed{(x-3)^2}$

Check: $x^2 - 3x - 3x + 9$

$= x^2 - 6x + 9$