Review of Factoring So far

Note Title 3/31/2015

Ex: Factor.

$32x^3 - 50xy^2$

GCF: $2x$

$= 2x(16x^2 - 25y^2)$

$= 2x((4x)^2 - (5y)^2)$

$= \boxed{2x(4x+5y)(4x-5y)}$

Recall: Difference of 2 squares

$a^2 - b^2 = (a+b)(a-b)$

Check: $(4x+5y)(4x-5y)$

$= 16x^2 - 20xy + 20xy - 25y^2$

$= 16x^2 - 25y^2$ ✓

Ex: Factor.

$-7x^2 + 10x - 3$

$= -1(7x^2 - 10x + 3)$

(+) same signs want a sum of 10x for middle term

$= \boxed{-1(7x-3)(x-1)}$

$= -(7x-3)(x-1)$

Check: $(7x-3)(x-1)$

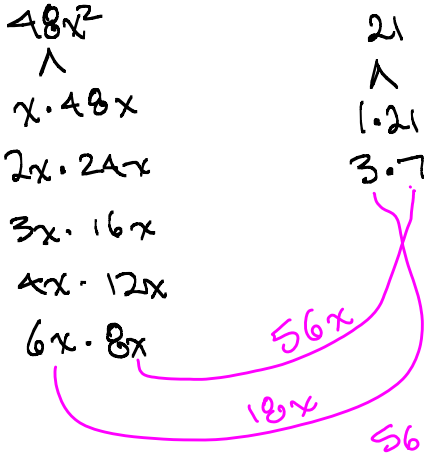
$= 7x^2 - 7x - 3x + 3$

$= 7x^2 - 10x + 3$ ✓

Ex: $48x^2 + 38x - 21$

(-) signs are opposite want difference of 38x for middle te

$\boxed{(6x+7)(8x-3)}$



Check: $48x^2 - 18x + 56x - 21$

$= 48x^2 + 38x - 21$ ✓ ok

56 − 18 = 38

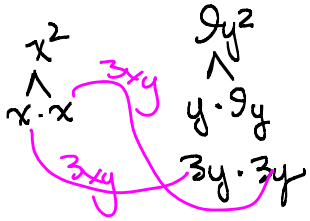
Ex: $3x^2 + 18xy + 27y^2$

$= 3(x^2 + 6xy + 9y^2)$

(+) same signs want sum of 6xy for middle term

$= 3(x+3y)(x+3y)$

Answer: $3(x+3y)^2$



Check: $(x+3y)(x+3y)$

$= x^2 + 3xy + 3xy + 9y^2 = x^2 + 6xy + 9y^2$

Ex: ~~Factor.~~

~~$16x^2 + 392xy + 49y^2$~~

~~Try: $(4x + 7y)^2$~~

Ex: Factor.

$16x^2 - 56xy + 49y^2$

Try: $(4x - 7y)^2$

Check: $(4x-7y)(4x-7y)$

$= 16x^2 - 28xy - 28xy + 49y^2$

$= 16x^2 - 56xy + 49y^2$ ✓

Answer: $\boxed{(4x-7y)^2}$

$1 \cdot 28$, $+28$, 56

Ex: $4x^2 - 20x + 9$

Try: $(2x - 3)^2$

Check: $(2x-3)(2x-3)$

$= 4x^2 - 6x - 6x + 9$

$= 4x^2 - 12x + 9$ No!

start over:

$4x^2 - 20x + 9$ (+) same signs, sum of $-20x$

$\boxed{(2x - 9)(2x - 1)}$

$= 4x^2 - 2x - 18x + 9$

$= 4x^2 - 20x + 9$ ✓

$4x^2$: $x \cdot 4x$, $2x \cdot 2x$; 9: $1 \cdot 9$, $3 \cdot 3$; $2x$, $18x$

Ex: Factor.

$x^4 - 13x^2 + 36$ (+) same signs, sum of $-13x^2$

$= (x^2 - 4)(x^2 - 9)$

$= \boxed{(x+2)(x-2)(x+3)(x-3)}$

x^4: $x^2 \cdot x^2$, ~~$x^3 \cdot x$~~; 36: $1 \cdot 36$, $2 \cdot 18$, $3 \cdot 12$, $4 \cdot 9$, $6 \cdot 6$; $-4x^2$, $-9x^2$

Check: $(x^2-4)(x^2-9)$

$= x^4 - 9x^2 - 4x^2 + 36$

$= x^4 - 13x^2 + 36$ ✓

5.5: Sum and Difference of Two Cubes

Know these formulas:

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$
$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$

* Sum and Difference of Two Cubes Factorizations

Mnemonic Device (for remembering the signs)

S	Same	M	Match
O	Opposite	O	Opposite
A	Always		
P	Positive	P	Positive

Perfect Cubes

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$

Ex: Factor.

$y^3 - 27$

$= y^3 - 3^3$

use $a^3-b^3 = (a-b)(a^2+ab+b^2)$ with $a=y$, $b=3$

$= (y-3)(y^2+3y+3^2)$

$= \boxed{(y-3)(y^2+3y+9)}$

Check: $y(y^2+3y+9) - 3(y^2+3y+9)$

$= y^3 + 3y^2 + 9y$
$\quad - 3y^2 - 9y - 27$
$= y^3 - 27$ ✓ ok

Ex: $8x^3 + 125$

$= (2x)^3 + (5)^3$

$= (2x + 5)((2x)^2 - 2x(5) + 5^2)$

$= \boxed{(2x+5)(4x^2 - 10x + 25)}$

need to square both the variable and the coefficient

Careful! Common Mistake: $(2x+5)(2x^2 - 10x + 25)$

Check: $(2x+5)(4x^2 - 10x + 25)$

$8x^3 - 20x^2 + 50x$

$+ 20x - 50x + 125$

$= 8x^3 + 125$ ✓

Ex: Factor.

$2x^4 + 5x^3 - 2x - 5$

$= (2x^4 + 5x^3) + (-2x - 5)$

$= x^3(2x+5) - 1(2x+5)$

It factors more! difference of 2 cubes

$= (2x+5)(x^3 - 1)$

$= (2x+5)(x^3 - 1^3)$

$= (2x+5)(x-1)(x^2 + 1x + 1^2)$

$= \boxed{(2x+5)(x-1)(x^2+x+1)}$

Section 5.6: Factoring: A general Review

→ work these problems on your own! Very, very important!

5.7: Solving Quadratic Equations by Factoring

A quadratic equation (in x) is an equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is standard form for a quadratic equation

quadratic term — linear term — constant term

Zero Product Property:

If the product of two numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB = 0$, then $A = 0$ or $B = 0$.

Ex: Solve.

$x^2 - 24 = 10x$

$-10x \quad -10x$

[write in standard form $ax^2 + bx + c = 0$]

$x^2 - 10x - 24 = 0$

(-) opp signs, want difference

[Factor it.]

$(x - 12)(x + 2) = 0$

[Set each factor equal to 0 (use Zero Product Property)]

OR

$x - 12 = 0$ | $x + 2 = 0$

$+12 \quad +12$ | $-2 \quad -2$

$x = 12$ | $x = -2$

Solution Set: $\{12, -2\}$

Remember:
expressions get simplified (or factored)
equations get solved

24
1 · 24
2 · 12
3 · 8
4 · 6

Check:

$(x-12)(x+2)$
$= x^2 + 2x - 12x - 24$
$= x^2 - 10x - 24$ ✓

Check: $x^2 - 24 = 10x$

$x = 12$: $12^2 - 24 = 10(12)$
$144 - 24 = 120$
$120 = 120$ ✓

Check: $x = -2$: $(-2)^2 - 24 = 10(-2)$
$4 - 24 = -20$
$-20 = -20$ ✓ ok

Ex: Solve.

$$4y^2 = 81$$

[write in std form, with one side 0]

$$4y^2 - 81 = 0$$

[Factor by Difference of 2 squares]

$$(2y)^2 - (9)^2 = 0$$

$$(2y+9)(2y-9) = 0$$

[Set each Factor = 0]

$2y+9=0$ OR $2y-9=0$

$2y = -9$ | $2y = 9$

$\frac{2y}{2} = \frac{-9}{2}$ | $\frac{2y}{2} = \frac{9}{2}$

$y = -\frac{9}{2}$ | $y = \frac{9}{2}$

Solution Set: $\left\{-\frac{9}{2}, \frac{9}{2}\right\}$

OR

$\left\{\pm\frac{9}{2}\right\}$

Read "plus or minus $\frac{9}{2}$"

Ex: Solve.

$$-2x^2 + 4x + 70 = 0$$

$$-2(x^2 - 2x - 35) = 0$$

$$-2(x-7)(x+5) = 0$$

$-2 = 0$ (never true) OR $x-7=0$ | $x+5=0$

$x = 7$ | $x = -5$

Solution Set: $\{7, -5\}$

Check

$(x-7)(x+5)$

$= x^2 + 5x - 7x - 35$

$= x^2 - 2x - 35$ ✓