5.7: Solving Quadratic Equations by Factoring (continued)

Note Title 4/2/2015

Steps for solving a quadratic equation by factoring:

1) Write in standard form $ax^2+bx+c=0$.
2) Factor the nonzero side. (Don't forget to check by multiplying!)
3) Set each factor equal to 0.
4) Solve the resulting linear equations.

Ex: Solve.

$x^2+60=19x$

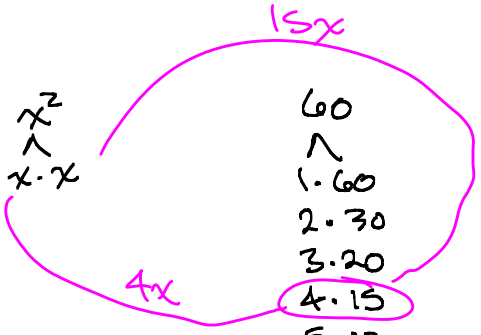
$x^2-19x+60=0$ (+) same signs want sum of $19x$

$(x-4)(x-15)=0$

$x-4=0$ OR $x-15=0$

$x=4$ $\quad x=15$

Solution Set: $\{4,15\}$ answer



Check: $(x-4)(x-15)$

$=x^2-15x-4x+60$

$=x^2-19x+60$ ✓

Check:

$x=4$: $4^2+60=19(4)$

$16+60=76$

$76=76$ ✓

$x=15$: $15^2+60=19(15)$

$225+60=285$

$285=285$ ✓

$15\times 4 = 60$; $3\times 19 = 4$; 76

$19\times 15 = 95$; $1\times 19 = 19$; 285

$15\times 15 = 15$; $15\times 15 = 225$

Ex: Solve.

$x^2=16$

$x^2-16=0$

$(x+4)(x-4)=0$

$x+4=0$ OR $x-4=0$

$x=-4$ $\quad x=4$

Check: $(x+4)(x-4)$

$=x^2-4x+4x-16$

$=x^2-16$ ✓

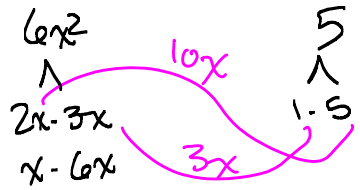
Solution Set: $\{4,-4\}$ or $\{\pm 4\}$ "plus or minus 4"

Ex: Solve.

$$6x^2 = 7x + 5$$

$$6x^2 - 7x - 5 = 0$$

(−) signs are opposite want a difference of 7x for middle term



$$(2x + 1)(3x - 5) = 0$$

$2x + 1 = 0$ OR $3x - 5 = 0$

$2x = -1$ | $3x = 5$

$\frac{2x}{2} = \frac{-1}{2}$ | $\frac{3x}{3} = \frac{5}{3}$

$x = -\frac{1}{2}$ | $x = \frac{5}{3}$

Sol'n Set: $\left\{-\frac{1}{2}, \frac{5}{3}\right\}$

Check:

$(2x+1)(3x-5)$

$= 6x^2 - 10x + 3x - 5$

$= 6x^2 - 7x - 5$ ✓

Ex: Solve.

$$12x^2 + 76x + 24 = 0$$

$$2(6x^2 + 38x + 12) = 0$$

$$2 \cdot 2(3x^2 + 19x + 6) = 0$$

$$4(3x^2 + 19x + 6) = 0$$

(+) same signs want sum of 19x for middle term

$$4(3x + 1)(x + 6) = 0$$

$4 = 0$ OR $3x + 1 = 0$ OR $x + 6 = 0$

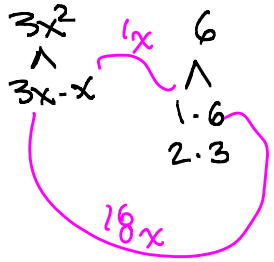
Never true | $3x = -1$ | $x = -6$

$\frac{3x}{3} = \frac{-1}{3}$

$x = -\frac{1}{3}$

Sol'n Set: $\left\{-\frac{1}{3}, -6\right\}$

$2 \overline{)76}$ → 38, 6, 16, 16, 0



Check:

$(3x+1)(x+6)$

$= 3x^2 + 18x + 1x + 6$

$= 3x^2 + 19x + 6$ ✓

Ex: Solve.

$$-14x^2 + 24x = -2x^3$$

$$2x^3 - 14x^2 + 24x = 0$$

(+) same signs sum of 7x

$$2x(x^2 - 7x + 12) = 0$$

$$2x(x - 4)(x - 3) = 0$$

$2x = 0$ OR $x - 4 = 0$ OR $x - 3 = 0$

$\frac{2x}{2} = \frac{0}{2}$ | $x = 4$ | $x = 3$

$x = 0$

Sol'n Set: $\{0, 3, 4\}$

Note: This is not a quadratic (degree 2) eqn.

This is a cubic eqn. (degree 3)

Check: $(x-4)(x-3)$

$= x^2 - 3x - 4x + 12$

$= x^2 - 7x + 12$

$2x(x^2 - 7x + 12)$

$= 2x^3 - 14x^2 + 24x$ ✓

Ex: $x^2 = 9x^4$

$x^2 - 9x^4 = 0$

$-9x^4 + x^2 = 0$

$-x^2(9x^2 - 1) = 0$

$-x^2(3x+1)(3x-1) = 0$

$-x^2 = 0 \mid 3x+1 = 0 \mid 3x-1 = 0$

$-x = 0 \mid x = 0 \mid 3x = -1 \mid 3x = 1$

$\frac{-x}{-1} = \frac{0}{-1} \mid x = -\frac{1}{3} \mid x = \frac{1}{3}$

$x = 0$

$9x^2 - 1$

$(3x)^2 - (1)^2$

$(3x+1)(3x-1)$

Solution set: $\{0, \pm\frac{1}{3}\}$ or $\{0, -\frac{1}{3}, \frac{1}{3}\}$

Note: could do it like this:

$x^2 = 9x^4$

$0 = 9x^4 - x^2$

$0 = x^2(9x^2 - 1)$

$0 = x^2(3x+1)(3x-1)$

$x = 0, \; x = -\frac{1}{3}, \; x = \frac{1}{3}$

5.8 Applications of Quadratic Equations (word problems!)

Ex: The length of a rectangle is 6" more than four times its width. The area of the rectangle is 70 square inches. Find the length and width.

length: $4x+6$

width: x

length $4x+6$ — compare to x — width x

Area of rectangle = (length)(width)

$70 = (4x+6)(x)$

$x(4x+6) = 70$

$4x^2 + 6x = 70$

$4x^2 + 6x - 70 = 0$ (−) signs are opposite, want difference of $3x$

$2(2x^2 + 3x - 35) = 0$

$2(2x - 7)(x + 5) = 0$

Cont'd next page

Note: Perimeter of rectangle = 2(length) + 2(width)

$2x^2$ ↑ $2x \cdot x$; 35 ↑ $1 \cdot 35$, $5 \cdot 7$; $10x$; $7x$

Check: $(2x-7)(x+5)$

$= 2x^2 + 10x - 7x - 35$

$= 2x^2 + 3x - 35$ ✓

Previous example cont'd:

$$2(2x-7)(x+5)=0$$

OR: $2=0$ — never true

OR: $2x-7=0$, $2x=7$, $x=\frac{7}{2}$

OR: $x+5=0$, $x=-5$ — Throw out.

Solutions to eqn are $\frac{7}{2}$ and -5.

But -5 does not make sense for the width (or length) of a rectangle.

Throw out -5.

width: $x=\frac{7}{2}=3\frac{1}{2}$ inches

length: $4x+6=4\left(\frac{7}{2}\right)+6=\frac{28}{2}+6=14+6=20$ inches

The width is $3\frac{1}{2}$" and the length is 20".

Check it: 1st sentence: 4 times width: $4\left(\frac{7}{2}\right)=\frac{28}{2}=14$

6" more: 20" ✓

2nd sentence: Area = length (width)

$=20"\left(\frac{7}{2}"\right)=\frac{140}{2}$ in^2

$=70$ in^2 ✓

Ex: The height of a triangle is 3 feet less than five times its base. The area of the triangle is 13 ft^2. Find the base and height.

five times its base: $5x$

base: x

height: $5x-3$

height compare to base: x

See next page.

Area of triangle $= \frac{1}{2}$ (base)(height)

$$13 = \frac{1}{2}(x)(5x-3)$$

$$\frac{1}{2}x(5x-3) = 13$$

multiply both sides by 2:

$$(2)\frac{1}{2}x(5x-3) = 13(2)$$

$$x(5x-3) = 26$$

$$5x^2 - 3x = 26$$

$$5x^2 - 3x - 26 = 0$$

$$(5x - 13)(x + 2) = 0$$

$5x - 13 = 0$ | $x + 2 = 0$

$5x = 13$ | $x = -2$

$x = \frac{13}{5}$ | Throw out! Negative number does not make sense for a dimension.

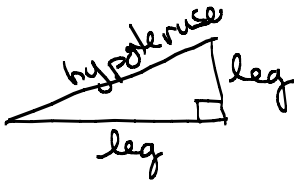
Base: $x = \frac{13}{5} = 2\frac{3}{5}$ ft

Height: $5x - 3 = 5\left(\frac{13}{5}\right) - 3 = 13 - 3 = 10$ ft

The base is $2\frac{3}{5}$ ft and the height is 10 ft.

Ex: The hypotenuse of a right triangle is 2" longer than the longer leg. The longer leg is 4" longer than twice the shorter leg. Find the lengths of all the sides. $2x$

Right Triangle:



Pythagorean Theorem: In a right triangle,

$$a^2 + b^2 = c^2 \quad (c = \text{hypotenuse})$$

hypotenuse: $(2x+4)+2 = 2x+6$

longer leg: $2x+4$

shorter leg: x

See next page

hypotenuse $\xrightarrow{\text{compare to}}$ longer leg $\xrightarrow{\text{compare to}}$ shorter leg

$2x+6$ | $2x+4$ | x

$a^2 + b^2 = c^2$ Pythagorean Theorem

$(x)^2 + (2x+4)^2 = (2x+6)^2$

$x^2 + (2x+4)(2x+4) = (2x+6)(2x+6)$

$x^2 + 4x^2 + 8x + 8x + 16 = 4x^2 + 12x + 12x + 36$

$5x^2 + 16x + 16 = 4x^2 + 24x + 36$

$-4x^2 \quad -24x \quad -36 \qquad -4x^2 \qquad -24x \quad -36$

$x^2 - 8x - 20 = 0$

$(x - 10)(x+2) = 0$

$x - 10 = 0$ | $x + 2 = 0$

$x = 10$ | $x = -2$

Throw out! negative number does not make sense for a dimension

Short leg: $x = 10'$

longer leg: $2x+4 = 2(10) + 4 = 24'$

hypotenuse: $2x + 6 = 2(10) + 6 = 26'$

The legs are 10 ft and 24 ft, and the hypotenuse is 26 ft.

Due Tuesday:

Preceding Assignment:

Read Section 6.1 (pp. 452-456).

Rework Example 3 (p. 454)

Work Matched Problem #3