5.8: Applications of Quadratic Equations (cont'd)

Note Title 4/7/2015

Ex.: Find two consecutive integers whose product is 27 more than 5 times the larger integer.

1st integer (smaller): x

2nd integer (larger): $x+1$

Product = 27 more than $\underbrace{5 \text{ times larger}}_{5(x+1)}$

$$x(x+1) = 5(x+1) + 27$$

$$x^2 + x = 5x + 5 + 27$$

$$x^2 + x = 5x + 32$$

$$-5x - 32 \quad -5x \quad -32$$

$$x^2 - 4x - 32 = 0$$

(−) signs are opposite want difference of $4x$

$$(x+4)(x-8) = 0$$

x^2: $x \cdot x$; 32: $1 \cdot 32$, $2 \cdot 16$, $4 \cdot 8$

$8x$ $4x$

Check: $(x+4)(x-8)$

$x^2 - 8x + 4x - 32$

$x^2 - 4x - 32$ ✓ok

$x + 4 = 0$ or $x - 8 = 0$

$x = -4$ | $x = 8$

$\boxed{x = -4}$ 1st integer: $x = -4$

2nd integer: $x + 1 = -4 + 1$
$= -3$

The two integers are -4 and -3.

$\boxed{x = 8}$ 1st integer: $x = 8$

2nd integer: $x + 1 = 8 + 1 = 9$

The two integers are 8 and 9.

The pair of integers is $-3, -4$ or $8, 9$.

Check answers:

$-3, -4$ | consecutive? Yes

Product: $(-3)(-4) = 12$ ✓ same!

5 times larger: $5(-3) = -15$

27 more: $-15 + 27 = 12$ ✓ok

$8, 9$ | consecutive? Yes

Product: $8(9) = 72$ ✓ same!

5 times larger: $5(9) = 45$

27 more: $45 + 27 = 72$

✓ok

Ex: (5.8 #6) The product of two consecutive odd integers is 1 less than 4 times their sum. Find the integers.

1st integer: x

2nd integer: $x+2$

Product = 1 less than 4 times sum

Product = 4(sum) − 1

$x(x+2) = 4(x+x+2) - 1$

$x^2+2x = 4(2x+2)-1$

$x^2+2x = 8x+8-1$

$x^2+2x = 8x+7$

$\quad -8x \quad -8x \quad -7$

$\quad -7$

$x^2-6x-7=0$

$(x-7)(x+1)=0$

$x-7=0$ or $x+1=0$

$x=7$ | $x=-1$

Check:

$(x-7)(x+1)$

$=x^2+1x-7x-7$

x^2-6x-7 ✓ok

$x=7$ | 1st integer: $x=7$

2nd integer: $x+2=7+2=9$

(The two integers are 7 and 9).

$x=-1$ | 1st integer: $x=-1$

2nd integer: $x+2=-1+2=1$

(The two integers are −1, 1).

$\boxed{\text{The pair of integers is either 7, 9 or −1, 1.}}$

Check it: 7, 9 | Consecutive odd integers? Yes

Product: $7(9)=63$

Sum: $7+9=16$

4 times sum: $4(16)=64$

1 less: 63

Same! ✓

−1, 1 | consecutive and odd? Yes

Product: $-1(1)=-1$

Sum: $-1+1=0$

4 times sum: $4(0)=0$

1 less: -1

Same! ✓

6.1: Simplifying Rational Expressions

Recall: Rational Number: any number that can be written as the ratio (fraction, quotient) of two integers.

In other words, it can be written as $\frac{a}{b}$, where a and b are integers.

Examples of Rational Numbers:

$\frac{2}{5}$, $-\frac{3}{2}$, 3, 0, 0.47, $0.\bar{3}$, $3.02\overline{615}$, 3.14

$3 = \frac{3}{1}$, $0 = \frac{0}{1}$, $0.47 = \frac{47}{100}$, $0.\bar{3} = 0.3333\ldots = \frac{1}{3}$, $3.02\overline{615} = 3.02615615\ldots$, $3.14 = \frac{314}{100}$

Examples of Irrational Numbers: (not rational)

π, $\sqrt{2}$, $\sqrt{3}$, e

Note: Decimals that end or repeat are rational. (They can be written as a ratio of 2 integers)

Rational Expression: Can be written as the ratio (fraction, quotient) of 2 polynomials.

Examples of rational expressions:

$$\frac{x^2+4}{x^5-8}, \quad \frac{x^2-x-12}{x^2-9}, \quad \frac{2}{x^2+5}, \quad \underbrace{x^{-1}+y^{-1}}_{\frac{1}{x}+\frac{1}{y}}$$

Not Rational Expressions

$$\frac{\sqrt{x^2+4}}{x-9}, \quad \sqrt{x^2+5}$$

Simplifying a rational expression: reducing the fraction by "canceling out" common factors.

Ex: Simplify $\frac{8}{28}$.

$$\frac{8}{28} = \frac{2\cdot 4}{4\cdot 7} = \frac{\cancel{4}^{1}}{\cancel{4}_{1}}\cdot\frac{2}{7} = 1\cdot\frac{2}{7} = \boxed{\frac{2}{7}}$$

Ex: Simplify.

$$\frac{\cancel{24}^{3}x^4y^3}{\cancel{32}_{4}x^5y} = \boxed{\frac{3y^2}{4x}}$$

Ex:
$$\frac{x^2+8x+15}{x^2+5x+6} = \frac{\cancel{(x+3)}^{1}(x+5)}{(x+2)\cancel{(x+3)}_{1}}$$

$$= \boxed{\frac{x+5}{x+2}}$$

Note: Restrictions: (cannot have 0 denominator)

$x \neq -2$

$x \neq -3$

Scratchwork

same (+) sum

$x^2+8x+15$

15

$1\cdot 15$

$3\cdot 5$

$(x+3)(x+5)$

$= x^2+5x+3x+15$

$= x^2+8x+15$ ✓

$(x+2)(x+3)$

$= x^2+3x+2x+6$

$= x^2+5x+6$ ✓

Ex: Simplify.

$$\frac{x-3}{x^2-7x+12} = \frac{\cancel{x-3}^{1}}{\cancel{(x-3)}_{1}(x-4)} = \boxed{\frac{1}{x-4}}$$

Restrictions: $x \neq 4$

$x \neq 3$

Ex: Simplify.

Recommended Method

$$\frac{x-2}{4-x^2} = \frac{x-2}{-x^2+4} = \frac{x-2}{-1(x^2-4)} = \frac{\cancel{x-2}^{\,1}}{-1(x+2)\cancel{(x-2)}_{\,1}}$$

$$= \frac{1}{-1(x+2)}$$

$$= \boxed{-\frac{1}{x+2}}$$

Could do this:

$$\frac{x-2}{4-x^2} = \frac{x-2}{(2+x)(2-x)} = \frac{x-2}{(2+x)(-1)(-2+x)}$$

$$= \frac{\cancel{x-2}^{\,1}}{-1(x+2)\cancel{(x-2)}_{\,1}} = \frac{1}{-1(x+2)} = \boxed{-\frac{1}{x+2}}$$

6.2: Multiplying and Dividing Rational Expressions

Ex: $\frac{2}{3} \cdot \frac{4}{5} = \boxed{\frac{8}{15}}$

Ex: $\frac{16}{15} \cdot \frac{21}{32} = \frac{\cancel{16}^{\,1}}{\cancel{15}_{\,5}} \cdot \frac{\cancel{21}^{\,7}}{\cancel{32}_{\,2}} = \boxed{\frac{7}{10}}$

Ex:
Simplify.

$$\frac{x^2+2x}{x^2-4} \cdot \frac{x^2-4x+4}{x^2-x} = \frac{\cancel{x}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}(x-2)}{\cancel{x}(x-1)}$$

$$= \boxed{\frac{x-2}{x-1}}$$

Ex:
& Simplify.

$$\frac{x^2-9}{x+5} \div \frac{x^2-7x+12}{x^2-25} = \frac{x^2-9}{x+5} \cdot \frac{x^2-25}{x^2-7x+12}$$

$$= \frac{(x+3)\cancel{(x-3)}}{\cancel{x+5}} \cdot \frac{\cancel{(x+5)}(x-5)}{\cancel{(x-3)}(x-4)} = \boxed{\frac{(x+3)(x-5)}{x-4}}$$