6.3: Adding and Subtracting Rational Expressions (cont'd)

Note Title 4/16/2015

Section 6.3

Add to Homework List:

6.3 # 23, 27, 31, 29, 33, 35, 37, 39

Ex: $\dfrac{2x+1}{x^2-9} + \dfrac{3x-5}{9-x^2}$

$= \dfrac{2x+1}{x^2-9} + \dfrac{3x-5}{9-x^2}\left(\dfrac{-1}{-1}\right)$

$= \dfrac{2x+1}{x^2-9} + \dfrac{-3x+5}{-9+x^2} = \dfrac{2x+1}{x^2-9} + \dfrac{-3x+5}{x^2-9}$

$= \dfrac{2x+1-3x+5}{x^2-9} = \boxed{\dfrac{-x+6}{x^2-9}} = \dfrac{-1(x-6)}{(x+3)(x-3)} = \boxed{-\dfrac{x-6}{(x+3)(x-3)}}$

Ex: $\dfrac{2}{x+2} - \dfrac{6x+4}{x^2-4}$

Factor all the denominators.

LCD: $(x+2)(x-2)$

$= \dfrac{2}{x+2} + \dfrac{-6x-4}{(x+2)(x-2)}$

$= \dfrac{2}{x+2}\left(\dfrac{x-2}{x-2}\right) + \dfrac{-6x-4}{(x+2)(x-2)}$

$= \dfrac{2x-4-6x-4}{(x+2)(x-2)} = \dfrac{-4x-8}{(x+2)(x-2)} = \dfrac{-4\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$

$= \boxed{\dfrac{-4}{x-2}} = \boxed{-\dfrac{4}{x-2}}$

Ex: Simplify.

$$\frac{2x-1}{x^2+x-6} - \frac{x+2}{x^2+5x+6}$$

LCD: $(x-2)(x+3)(x+2)$

$$= \frac{2x-1}{(x-2)(x+3)} + \frac{-1(x+2)}{(x+2)(x+3)}$$

$$= \frac{2x-1}{(x-2)(x+3)}\left(\frac{x+2}{x+2}\right) + \frac{-1(x+2)}{(x+2)(x+3)}\left(\frac{x-2}{x-2}\right)$$

$$= \frac{2x^2+3x-2-1(x^2-4)}{(x-2)(x+3)(x+2)} = \frac{2x^2+3x-2-x^2+4}{(x-2)(x+3)(x+2)}$$

$$= \frac{x^2+3x+2}{(x-2)(x+3)(x+2)} = \frac{\cancel{(x+2)}(x+1)}{(x-2)(x+3)\cancel{(x+2)}} = \boxed{\frac{x+1}{(x-2)(x+3)}}$$

Ex: Simplify.

$$\frac{4x+1}{x^2-3x} - \frac{3x}{x^2-x-6}$$

Factor all denominators.

LCD: $x(x-3)(x+2)$

$$= \frac{4x+1}{x(x-3)} + \frac{-3x}{(x-3)(x+2)}$$

$$= \frac{4x+1}{x(x-3)}\left(\frac{x+2}{x+2}\right) + \frac{-3x}{(x-3)(x+2)}\left(\frac{x}{x}\right)$$

$$= \frac{4x^2+8x+1x+2-3x^2}{x(x-3)(x+2)} = \boxed{\frac{x^2+9x+2}{x(x-3)(x+2)}}$$

6.4: Solving Equations with Rational Expressions

Ex: Solve.

$$\frac{x}{5}+\frac{1}{5}=\frac{4}{5}$$

By inspection: $x=3$

Sol'n Set: $\{3\}$.

Notice: $\frac{x}{5}+\frac{1}{5}=\frac{4}{5}$.

If I multiply both sides by 5, the fractions will be gone.

$$\frac{5}{1}\left(\frac{x}{5}+\frac{1}{5}\right)=\left(\frac{4}{\cancel{5}}\right)\left(\frac{\cancel{5}}{1}\right)$$

$$\frac{\cancel{5}}{1}\left(\frac{x}{\cancel{5}}\right)+\frac{\cancel{5}}{1}\left(\frac{1}{\cancel{5}}\right)=4$$

$$x+\frac{1}{1}=\frac{4}{1}$$

$$x=3$$

Sol'n Set: $\boxed{\{3\}}$

Ex: Solve.

$$\frac{x}{5}-\frac{4x}{3}=\frac{1}{15}$$

LCD: 15

multiply both sides by 15

$$\frac{\overset{3}{\cancel{15}}}{1}\left(\frac{x}{\cancel{5}}\right)-\frac{4x}{\cancel{3}}\left(\frac{\overset{5}{\cancel{15}}}{1}\right)=\frac{1}{\cancel{15}}\left(\frac{\cancel{15}}{1}\right)$$

$$3x-20x=1$$

$$-17x=1$$

$$\frac{-17x}{-17}=\frac{1}{-17}$$

$$x=-\frac{1}{17}$$

Sol'n Set: $\boxed{\left\{-\frac{1}{17}\right\}}$

Ex: Solve.

$$\frac{2x-1}{2} - \frac{4x+3}{3} = \frac{x+5}{6}$$

LCD: 6
Multiply both sides by 6.

$$\left(\frac{\cancel{6}^{\,3}}{1}\right)\frac{2x-1}{\cancel{2}_{\,1}} + \frac{-4x-3}{\cancel{3}_{\,1}}\left(\frac{\cancel{6}^{\,2}}{1}\right) = \frac{x+5}{\cancel{6}_{\,1}}\left(\frac{\cancel{6}^{\,1}}{1}\right)$$

$$3(2x-1) + (-4x-3)(2) = x+5$$

$$6x-3-8x-6 = x+5$$

$$\underset{-x}{-2x} - 9 = \underset{-x}{x} + 5$$

$$-3x - \underset{+9}{9} = \underset{+9}{5}$$

$$-3x = 14$$

$$\frac{-3x}{-3} = \frac{14}{-3}$$

$$x = -\frac{14}{3}$$

$\boxed{\text{Sol'n Set: } \left\{-\frac{14}{3}\right\}}$

Ex: Solve.

$$\frac{1}{3x} - \frac{1}{6} = \frac{5}{x}$$

LCD: $6x$
Multiply both sides by $6x$.

$$\left(\frac{\cancel{6x}^{\,2}}{1}\right)\frac{1}{\cancel{3x}_{\,1}} + \frac{-1}{\cancel{6}}\left(\frac{\cancel{6}x}{1}\right) = \frac{5}{\cancel{x}}\left(\frac{6\cancel{x}}{1}\right)$$

$$\underset{-2}{2} - x = \underset{-2}{30}$$

$$-x = 28$$

$$\frac{-x}{-1} = \frac{28}{-1}$$

$$x = -28$$

$\boxed{\text{Sol'n Set: } \{-28\}}$

Note: $x \neq 3$

Ex: Solve.

$$\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$$

LCD: $2(x-3)$
Multiply both sides (all fractions) by $2(x-3)$.

$$\frac{(2)\cancel{(x-3)}}{1}\cdot\frac{2x}{\cancel{x-3}} + \frac{-1}{\cancel{2}}\cdot\frac{\cancel{(2)}(x-3)}{1} = \frac{6}{\cancel{x-3}}\cdot\frac{(2)\cancel{(x-3)}}{1}$$

$$4x - (x-3) = 12$$

See next page

Previous example cont'd:

$$4x - x + 3 = 12$$

$$3x + 3 = 12$$

$$3x = 9$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

Check: $\frac{2x}{x-3} - \frac{1}{2} = \frac{6}{x-3}$

$x=3$ $\frac{2(3)}{3-3} - \frac{1}{2} = \frac{6}{3-3}$

$\frac{6}{0} - \frac{1}{2} = \frac{6}{0}$ Division by 0 is undefined!

$x = 3$ does not check when substituted into the original equation.

Throw out 3.

$\boxed{\text{No Solution}}$

The value 3 is called an extraneous solution to the equation (not a solution at all).

Extraneous solution: Value generated by a correct solution process that does not check when substituted into the original equation.

Important: When solving an equation with variables in the denominator, you must check that the solutions do not result in 0 denominators.

Ex: Solve.

$$\frac{x+2}{x+3} = \frac{x-4}{x-2}$$

Restrictions: $x \neq 2$

$x \neq -3$

LCD: $(x+3)(x-2)$

Multiply both sides by the LCD.

$$\cancel{(x+3)}(x-2)\,\frac{x+2}{\cancel{x+3}} = \frac{x-4}{\cancel{x-2}}\,(x+3)\cancel{(x-2)}$$

$$(x-2)(x+2) = (x-4)(x+3)$$

$$x^2 + 2x - 2x - 4 = x^2 + 3x - 4x - 12$$

$$\cancel{x^2} - 4 = \cancel{x^2} - x - 12$$

$$-x^2 \qquad -x^2$$

$$-4 = -x - 12$$

$$+12 \qquad +12$$

$$8 = -x$$

$$\frac{8}{-1} = \frac{-x}{-1}$$

$$-8 = x$$

$\boxed{\text{Sol'n Set: } \{-8\}}$

Does $x = -8$ results in 0 denominators in original equation? No.