7.1 : Solving Linear Systems by Graphing

Note Title 4/21/2015

<u>System of equations:</u> group of equations

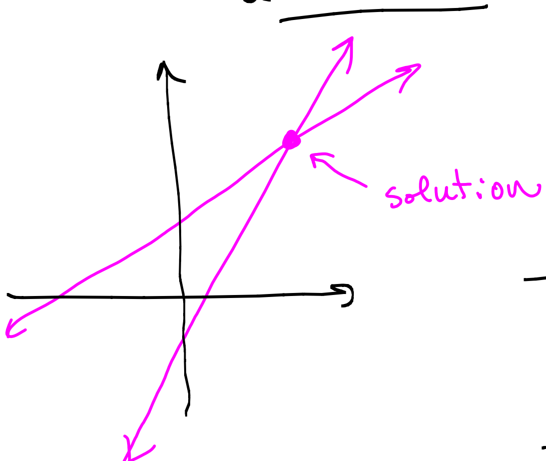
A <u>Linear System</u> is a system of linear equations.
(each equation can be written as
$Ax + By = C$, $Ax + By + Cz = D$

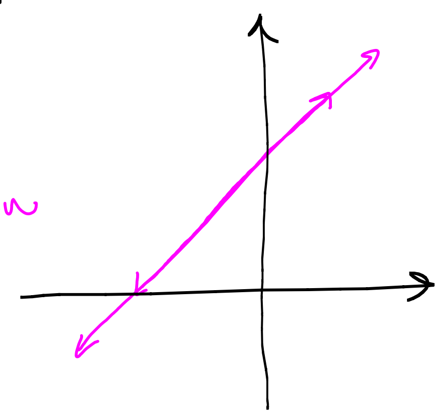
<u>A solution to a system of equations:</u> A set of values for the variables that makes <u>all</u> the equations true.

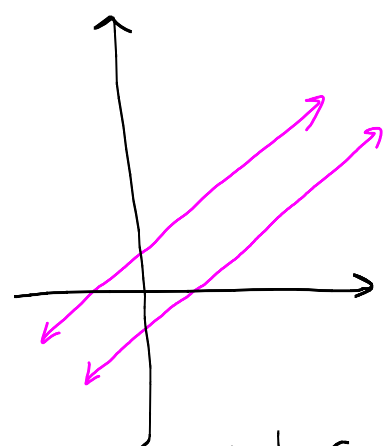
<u>For us:</u> we will solve systems of 2 equations in 2 variables (generally x and y).

<u>Ex:</u> $\left\{ \begin{array}{l} 2x - 5y = 4 \\ x + 6y = -10 \end{array} \right\}$ graphs of these equations are <u>lines</u>

<u>3 possible scenarios for a system of 2 equations in 2 variables:</u>







<u>Independent System</u>
One solution
(lines intersect (cross) at a single point)

<u>Dependent System</u>
Infinitely many solutions
(both equations represent the <u>same</u> <u>line</u>)

<u>Inconsistent System</u>
No Solution
(lines are parallel)

Question on Test #5

For an inconsistent, dependent, or independent system.

1) State the number of solutions
2) Describe the graph in words (lines are parallel, lines are the same, lines intersect at 1 point)
3) Illustrate with an example graph.

__Example:__ Is the ordered pair $(9,-2)$ a solution of this system?

$$\begin{cases} 2x + 5y = 8 \\ 3x - 2y = 23 \end{cases}$$

Put $x = 9,\ y = -2$ into $2x + 5y = 8$:

$$2(9) + 5(-2) = 8$$
$$18 - 10 = 8$$
$$8 = 8 \quad \text{True } \checkmark$$

So the ordered pair is a solution to the 1st equation.

Put $x = 9,\ y = -2$ into $3x - 2y = 23$.

$$3(9) - 2(-2) = 23$$
$$27 + 4 = 23$$
$$31 = 23 \quad \text{False.}$$

So $(9,-2)$ is not a solution to the 2nd equation.

$\boxed{\text{No, } (9,-2) \text{ is } \underline{\text{not}} \text{ a solution to the system.}}$

(To be a solution to the system, it must be a solution to __both__ of the equations)

Ex.: Is $\left(1, -\frac{2}{3}\right)$ a solution of this system?

$$\begin{cases} 2x - 3y = 4 \\ 5x + 6y = 1 \end{cases}$$

Put $x = 1$, $y = -\frac{2}{3}$ into $2x - 3y = 4$

$$2(1) - \cancel{3}^{1}\left(-\frac{2}{\cancel{3}_{1}}\right) = 4$$

$$2 + 2 = 4$$

$$4 = 4 \checkmark$$

Put $x = 1$, $y = -\frac{2}{3}$ into $5x + 6y = 1$

$$5(1) + \cancel{6}^{2}\left(-\frac{2}{\cancel{3}_{1}}\right) = 1$$

$$5 - 4 = 1$$

$$1 = 1 \checkmark$$

$\boxed{\text{Yes, } \left(1, -\frac{2}{3}\right) \text{ is a solution.}}$

Methods for solving linear systems:

1) Graphing (section 7.1)
2) Substitution (section 7.2)
3) Elimination (section 7.3)
4) Matrix Methods (College Algebra or Finite Math)

To solve a system by graphing:

Graph both lines.
Estimate the intersection point.
Check your solutions.

Example: Solve the system by graphing.

$$\begin{cases} x + 2y = 8 \\ x - 3y = 3 \end{cases}$$

1st, graph the line $x + 2y = 8$

Find the x-intercept: set $y = 0$: $x + 2(0) = 8$

$x = 8$ ordered pair $(8, 0)$

Find the y-intercept: Set $x=0$: $0+2y=8$

$$2y=8$$
$$y=4$$

ordered pair $(0,4)$

Next, graph the 2nd line $x-3y=3$:

Find the x-intercept: Set $y=0$: $x-3(0)=3$

$$x-0=3$$
$$x=3$$

ordered pair: $(3,0)$

Find the y-intercept: Set $x=0$: $0-3y=3$

$$-3y=3$$
$$\frac{-3y}{-3}=\frac{3}{-3}$$
$$y=-1$$

ordered pair $(0,-1)$

See graph paper

From graph, the solution appears to be $(6,1)$.

$x+2y=8$

$x=6, y=1 \Rightarrow 6+2(1)=8$

$$6+2=8$$
$$8=8$$

$x-3y=3$

$x=6, y=1 \Rightarrow 6-3(1)=3$

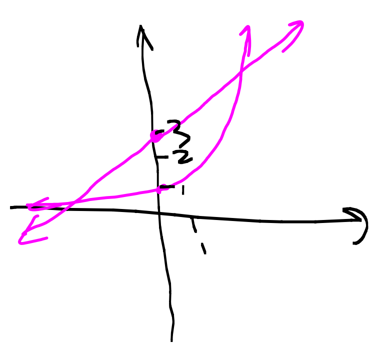
$$6-3=3$$
$$3=3\checkmark$$

$\boxed{\text{Solution is } (6,1).}$

You need to understand the concept of solving by graphing, because some systems (not linear systems) that cannot be solved with substitution or elimination.

Example: $y=2^x$

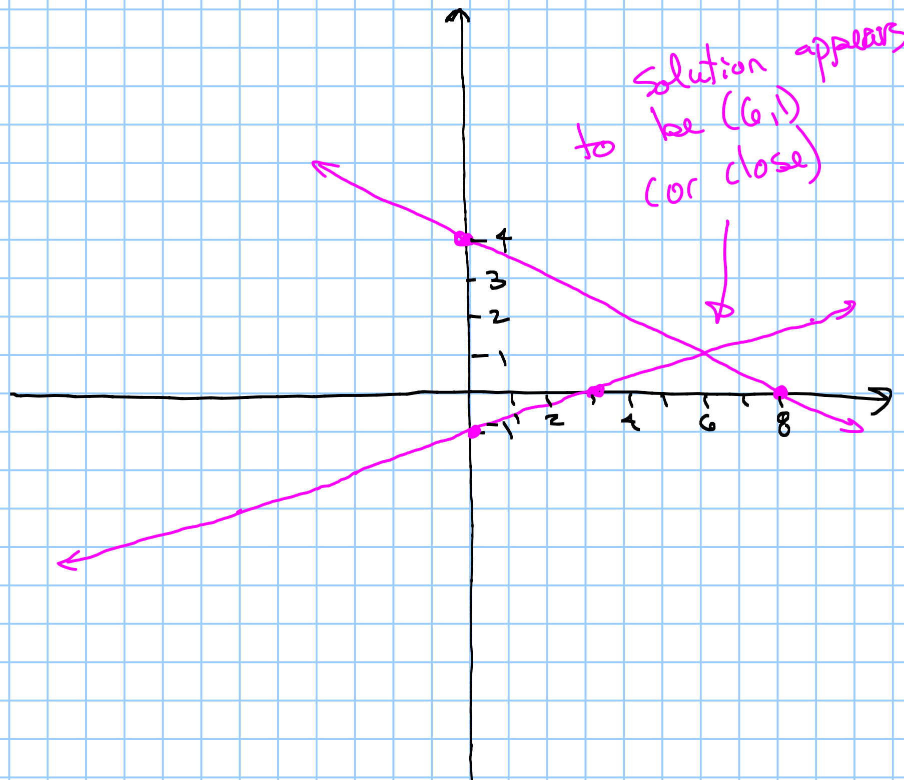
$y=x+3$



Note Title 2/5/2015

Example from Section 7.1

1st line	2nd line
(8,0)	(3,0)
(0,4)	(0,-1)



7.2: The Substitution Method (for solving linear systems)

Solving by substitution (step-by-step):

1) Solve one equation (your choice) for one variable (your choice).
2) Substitute this result into the other equation.
3) Solve for the remaining variable.
4) Substitute this value into either equation and solve.
5) Check your solution. (substitute into both equations and see if it makes them true)

Example: Solve the system.

$$\begin{cases} x - y = 9 \\ 2x - 11y = -18 \end{cases}$$

1) Solve $x - y = 9$ for x:

$$x = 9 + y$$

2) Substitute $x = 9 + y$ into $2x - 11y = -18$.

$$2(9 + y) - 11y = -18$$

3) Solve for y:

$$18 + 2y - 11y = -18$$

$$\begin{aligned} 18 - 9y &= -18 \\ {\scriptstyle -18} \quad &\quad {\scriptstyle -18} \end{aligned}$$

$$-9y = -36$$

$$\frac{-9y}{-9} = \frac{-36}{-9}$$

$$y = 4$$

4) Substitute $y = 4$ into $x - y = 9$: $x - 4 = 9$

$$x = 13$$

$\boxed{\text{Solution is } (13, 4).}$ Check is on next page.

5) Check it!

$x - y = 9$

$x = 13, y = 4 \Rightarrow 13 - 4 = 9$

$9 = 9$ ✓

$2x - 11y = -18$

$x = 13, y = 4 \Rightarrow 2(13) - 11(4) = -18$

$26 - 44 = -18$

$-18 = -18$ ✓

$\begin{array}{r} 44 \\ -26 \\ \hline 18 \end{array}$ ✓

Example: Solve the system by substitution.

$$\begin{cases} -4x + 3y = 19 \\ -6x - 5y = 0 \end{cases}$$

1) Solve $-6x - 5y = 0$ for y.

$\quad\; {+6x} \qquad\quad {+6x}$

$$-5y = 6x$$

$$\frac{-5y}{-5} = \frac{6x}{-5}$$

$$y = -\frac{6}{5}x$$

2) Substitute $y = -\frac{6}{5}x$ into the other equation:

$$-4x + 3y = 19$$

$$-4x + 3\left(-\frac{6}{5}x\right) = 19$$

$$-4x - \frac{18}{5}x = 19$$

$$-\frac{4x}{1}\left(\frac{5}{5}\right) - \frac{18}{5}x = 19$$

$$-\frac{20}{5}x - \frac{18}{5}x = 19$$

$$-\frac{38}{5}x = 19$$

$$\left(-\frac{5}{38}\right)\left(-\frac{38}{5}\right)x = \cancel{19}^{1}\left(-\frac{5}{\cancel{38}_{2}}\right)$$

$$x = \frac{-5}{2}$$

$\begin{array}{r} 19 \\ 2\overline{)38} \\ \underline{2}\;\; \\ 18 \\ \underline{18} \\ 0 \end{array}$

4) Put $x = -\frac{5}{2}$ into either equation

$$-6x - 5y = 0$$

$$-\frac{6}{1}\left(-\frac{5}{2}\right) - 5y = 0$$

$$\frac{30}{2} - 5y = 0$$

$$15 - 5y = 0$$
$$-15 \qquad -15$$

$$-5y = -15$$

$$\frac{-5y}{-5} = \frac{-15}{-5}$$

$$y = 3$$

$\boxed{\text{Solution is } \left(-\frac{5}{2}, 3\right).}$

check it!

5) Check:

$x = -\frac{5}{2},\ y = 3 \Rightarrow$

$$-6x - 5y = 0$$
$$-6\left(-\frac{5}{2}\right) - 5(3) = 0$$
$$15 - 15 = 0$$
$$0 = 0 \checkmark$$

$$-4x + 3y = 19$$
$$-4\left(-\frac{5}{2}\right) + 3(3) = 19$$
$$\frac{20}{2} + 9 = 19$$
$$10 + 9 = 19$$
$$19 = 19 \checkmark$$