7.3: The Elimination Method (for solving linear systems)

Note Title 4/23/2015

Methods for solving linear systems

1) Graphing (7.1)
2) Substitution (7.2)
3) Elimination (7.3)
4) Matrix Methods (College Algebra, Finite Math)

The Elimination Method (steps)

1) Multiply both sides of one or both equations by a strategic number.
2) Add the equations to eliminate a variable
3) Solve for the remaining variable.
4) Either
   a) Put that value into either equation and solve.
   or b) Do elimination (steps 1-3) again, to solve for the other variable.
5) Check your solution!

Ex: Solve the system by elimination.

$$5x - 12y = 1$$
$$6y - x = -2$$

Rewrite so the variables are in the usual order:

$$\begin{aligned} 5x - 12y &= 1 \longrightarrow & 5x - 12y &= 1 \\ -x + 6y &= -2 \xrightarrow{(2)} & -2x + 12y &= -4 \end{aligned}$$

Add: $3x + 0y = -3$

$3x + 0 = -3$

$\frac{3x}{3} = \frac{-3}{3}$

$x = -1$

See Next page

Example cont'd from previous page:

Put $x = -1$ into either of the original equations:

$$-x + 6y = -2$$

$$x = -1 \Rightarrow -(-1) + 6y = -2$$

$$\underset{-1}{1} + 6y = \underset{-1}{-2}$$

$$6y = -3$$

$$\frac{6y}{6} = \frac{-3}{6}$$

$$y = -\frac{1}{2}$$

Solution: $\boxed{\left(-1, -\frac{1}{2}\right)}$

Check it: $5x - 12y = 1$

$$x = -1, y = -\frac{1}{2} \Rightarrow 5(-1) - 12\left(-\frac{1}{2}\right) = 1$$

$$-5 + \frac{12}{2} = 1$$

$$-5 + 6 = 1$$

$$1 = 1 \checkmark$$

$$6y - x = -2$$

$$x = -1, y = -\frac{1}{2} \Rightarrow \cancelto{3}{6}\left(-\frac{1}{\cancel{2}}\right) - (-1) = -2$$

$$-3 + 1 = -2$$

$$-2 = -2 \checkmark$$

Example: Solve by elimination.

$$\begin{cases} 11x - 5y = 2 \\ 3x - y = 1 \end{cases}$$

$$11x - 5y = 2 \longrightarrow 11x - 5y = 2$$

$$3x - y = 1 \xrightarrow{(-5)} -15x + 5y = -5$$

Add: $-4x + 0 = -3$

$$-4x = -3$$

$$\frac{-4x}{-4} = \frac{-3}{-4}$$

$$x = \frac{3}{4}$$

You could put $x = \frac{3}{4}$ into one of the eqns, or you can do elimination again to find y. :

$$11x - 5y = 2 \xrightarrow{(-3)} -33x + 15y = -6$$

$$3x - y = 1 \xrightarrow{(11)} 33x - 11y = 11$$

Add: $0 + 4y = 5$

$$4y = 5$$

$$\frac{4y}{4} = \frac{5}{4}$$

$$y = \frac{5}{4}$$

$\boxed{\text{The solution is } \left(\tfrac{3}{4}, \tfrac{5}{4}\right).}$

Check:

$11x - 5y = 2$

$x = \frac{3}{4},\ y = \frac{5}{4} \Rightarrow 11\left(\frac{3}{4}\right) - 5\left(\frac{5}{4}\right) = 2$

$$\frac{33}{4} - \frac{25}{4} = 2$$

$$\frac{8}{4} = 2$$

$$2 = 2 \checkmark$$

$3x - y = 1$

$x = \frac{3}{4},\ y = \frac{5}{4} \Rightarrow 3\left(\frac{3}{4}\right) - \frac{5}{4} = 1$

$$\frac{9}{4} - \frac{5}{4} = 1$$

$$\frac{4}{4} = 1$$

$$1 = 1 \checkmark$$

Ex: $-7x - 9y = 7$

$-5x + 8y = 12$

$$\begin{aligned}
-7x - 9y = 7 &\xrightarrow{(8)} -56x - 72x = 56 \\
-5x + 8y = 12 &\xrightarrow{(9)} -45x + 72y = 108
\end{aligned}$$

Add: $-101x = 164$

$$\frac{-101x}{-101} = \frac{164}{-101}$$

$$x = -\frac{164}{101}$$

$$\begin{array}{r} \scriptstyle 1 \\ 12 \\ \times 9 \\ \hline 108 \end{array}$$

$$\begin{aligned}
-7x - 9y = 7 &\xrightarrow{(-5)} 35x + 45y = -35 \\
-5x + 8y = 12 &\xrightarrow{(7)} -35x + 56y = 84
\end{aligned}$$

Add: $101y = 49$

$$y = \frac{49}{101}$$

$$\begin{array}{r} 84 \\ -35 \\ \hline 49 \end{array}$$

$$\boxed{\text{The Solution is } \left(-\frac{164}{101}, \frac{49}{101}\right).}$$

Ex: Solve the system.

$$\begin{cases} -x + 6y = 2 \\ 3x - 18y = 5 \end{cases}$$

$$\begin{aligned}
-x + 6y = 2 &\xrightarrow{(3)} -3x + 18y = 6 \\
3x - 18y = 5 &\longrightarrow 3x - 18y = 5
\end{aligned}$$

Add: $0x + 0y = 11$

$0 = 11$ False statement.

The system is inconsistent.

$\boxed{\text{No Solution.}}$

Ex: Solve the system.

$$2x - 10y = 6$$
$$-5x + 25y = -15$$

$$2x - 10y = 6 \xrightarrow{(5)} 10x - 50y = 30$$
$$-5x + 25y = -15 \xrightarrow{(2)} -10x + 50y = -30$$

Add: $0x + 0y = 0$

$0 = 0$ True.

The system is $\boxed{\text{dependent.}}$

(has infinitely many solutions)

(lines are the same)

You really should describe the solutions. (You don't need to describe them, just write "dependent")

$$2x - 10y = 6$$
$$-10y = -2x + 6$$
$$\frac{-10y}{-10} = \frac{-2x}{-10} + \frac{6}{-10}$$
$$y = \frac{1}{5}x - \frac{3}{5}$$

Solution Set: $\{(x,y) \mid x = \text{any real \#},\ y = \frac{1}{5}x - \frac{3}{5}\}$

Summary: How to recognize inconsistent and dependent systems

When solving a system by elimination or by substitution, and both variables disappear:

1) If you get a false statement (such as $0 = 11$ or $5 = -6$, the system is inconsistent and has no solution. (lines are parallel)

2) If you get a true statement (such as $0 = 0$ or $5 = 5$) then the system is dependent and has infinitely many solutions. (lines are the same)

Hint: If you have fractions:

$$\begin{cases} \frac{x}{12} - \frac{y}{15} = -\frac{3}{20} \\ -\frac{x}{6} + \frac{y}{8} = \frac{5}{24} \end{cases}$$

1st get rid of fractions by multiplying by their LCD:

$$\frac{x}{12} - \frac{y}{15} = -\frac{3}{20} \xrightarrow{(60)} \overset{5}{\cancel{60}}\left(\frac{x}{\cancel{12}}\right) - \overset{4}{\cancel{60}}\left(\frac{y}{\cancel{15}}\right) = -\frac{3}{\cancel{20}}(\overset{3}{\cancel{60}})$$

$$5x - 4y = -9 \quad \text{New 1st eqn}$$

$$-\frac{x}{6} + \frac{y}{8} = \frac{5}{24} \xrightarrow{(24)} \overset{4}{\cancel{24}}\left(-\frac{x}{\cancel{6}}\right) + \overset{3}{\cancel{24}}\left(\frac{y}{\cancel{8}}\right) = \frac{5}{\cancel{24}}(\cancel{24})$$

$$-4x + 3y = 5$$

New system:
$$\begin{cases} 5x - 4y = -9 \\ -4x + 3y = 5 \end{cases}$$ then solve

7.4: Applications of Linear Systems

The difference of two positive numbers is 8.

The larger number is "=" 7 less than twice the smaller number.

larger number: x

smaller number: y

2 variables, so we need 2 equations.

1st sentence: $x - y = 8$

2nd sentence: larger # $= 2(\text{smaller}) - 7$

$$x = 2y - 7$$

System of 2 eqns: $\begin{cases} x - y = 8 \\ x = 2y - 7 \end{cases}$

Substitution: Substitute $x = 2y - 7$ into $x - y = 8$.

$$(\underbrace{2y-7}_{x}) - y = 8$$

$$2y - 7 - y = 8$$

$$y - 7 = 8$$

$$y = 15$$

Put $y = 15$ into $x - y = 8$:

$$x - 15 = 8$$

$$+15 \quad +15$$

$$x = 23$$

$\boxed{\text{The numbers are 15 and 23.}}$

Check against the words of the original problem:

1st sentence: Difference is 8? Yes. $23 - 15 = 8$

2nd sentence: Twice the smaller number: $2(15) = 30$

7 less: 23 ✓ OK

Ex.: Mike has $1.55 in dimes and nickels. He has 7 more nickels than dimes. How many of each does he have?

number of dimes: d
number of nickels: n

Need 2 equations.

1st sentence: (value of coins)

$$\$0.10\,d + \$0.05\,n = \$1.55$$

multiply by 100 to clear the decimals: $10d + 5n = 155$

2nd sentence: # nickels = # dimes + 7

$$n = d + 7$$

System $\begin{cases} 10d + 5n = 155 \\ n = d + 7 \end{cases}$

Substitute $n = d + 7$ into $10d + 5n = 155$

$$10d + 5(d+7) = 155$$
$$10d + 5d + 35 = 155$$
$$15d + 35 = 155$$
$$-35 \qquad -35$$
$$15d = 120$$
$$\frac{15d}{15} = \frac{120}{15}$$
$$d = 8$$

$$\begin{array}{r} {}^{3}15 \\ \times\ 7 \\ \hline 105 \end{array} \qquad \begin{array}{r} {}^{4}15 \\ \times\ 8 \\ \hline 120 \end{array}$$

number of dimes: $d = 8$.

Put $d = 8$ into $n = d + 7$
$n = 8 + 7 = 15$
# of nickels.

$\boxed{\text{He has 8 dimes and 15 nickels.}}$

Check against the words of the original problems

1st sentence: 8 dimes: $0.80
15 nickels: $0.75
$1.55 ✓

$$\begin{array}{r} {}^{2}15 \\ 5 \\ \hline 75 \end{array}$$

2nd sentence: 7 more nickels than dimes? Yes ✓