2.7: Linear Inequalities (cont'd)

Note Title 1/29/2015

Additive Property of Inequality

If $A < B$, then $A + C < B + C$.

This means we can add (or subtract) the same expression on both sides.

Example: Solve and graph.

$$x + 5 \leq 12$$
$$-5 \quad -5$$
$$x \leq 12 - 5$$
$$\boxed{x \leq 7}$$

Multiplicative Property of Inequality

If $A < B$ and $C > 0$, then $AC < BC$.

If $A < B$ and $C < 0$, then $A C > BC$.

This means:

We can multiply (or divide) both sides by the same positive number.

We can multiply (or divide) both sides by the same negative number if we reverse the inequality sign.

Why?

Start with $1 < 2$. It's true.

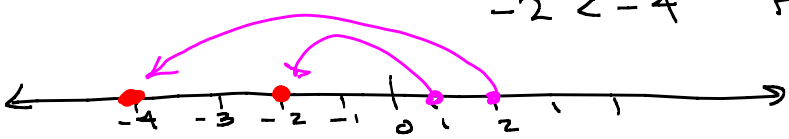
multiply both sides by 2: $1(2) < 2(2)$.

$2 < 4$. Still True.

Start with $1 < 2$ again. Still True.

multiply both sides by -2: $1(-2) < 2(-2)$ False.

$-2 < -4$ False!

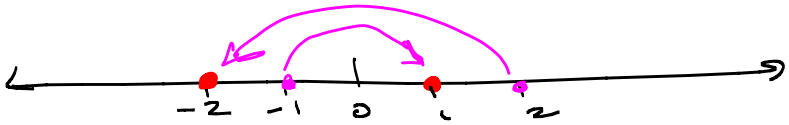


multiplying by a negative changes the order

Start with $-1 < 2$. True

Divide by -1: $\frac{-1}{-1} < \frac{2}{-1}$ False

$1 < -2$ False



Ex: Solve $3x \geq 18$

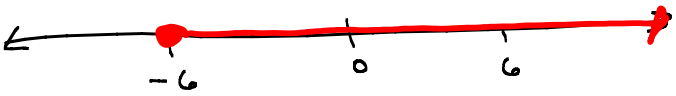
$\frac{3x}{3} \geq \frac{18}{3}$

$\boxed{x \geq 6}$

Ex: Solve $3x \geq -18$

$\frac{3x}{3} \geq \frac{-18}{3}$ ← Don't reverse the inequality, because we divided by a positive

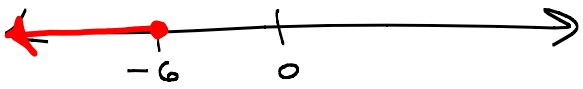
$\boxed{x \geq -6}$



Ex: Solve $-3x \geq 18$

$\frac{-3x}{-3} \leq \frac{18}{-3}$ ← Reverse the inequality sign!

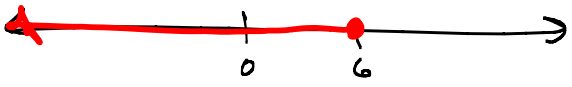
$\boxed{x \leq -6}$



Ex: Solve $-3x \geq -18$

$\frac{-3x}{-3} \leq \frac{-18}{-3}$ ← Reverse the inequality sign!

$\boxed{x \leq 6}$



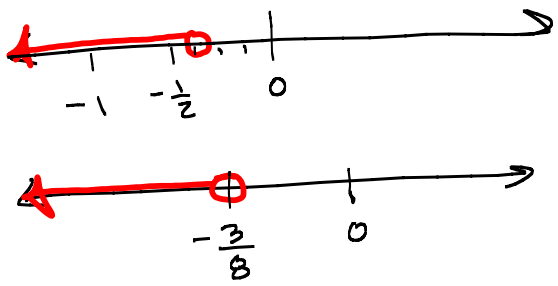
Ex: $\frac{2}{3}x < -\frac{1}{4}$ Solve and graph.

$$\frac{\frac{2}{3}x}{\frac{2}{3}} < \frac{-\frac{1}{4}}{\frac{2}{3}}$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) < -\frac{1}{4}\left(\frac{3}{2}\right)$$

$$1x < -\frac{3}{8}$$

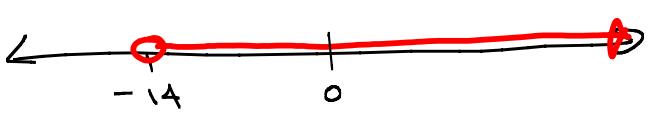
$$\boxed{x < -\frac{3}{8}}$$



Ex: $-\frac{1}{2}x < 7$

$$\left(-\frac{2}{1}\right)\left(-\frac{1}{2}x\right) > \frac{7}{1}\left(-\frac{2}{1}\right)$$ ← Reverse the inequality sign

$$\boxed{x > -14}$$



Ex: Solve. $3x + 7 < 5x - 5$
$\quad\quad -7 \quad\quad\quad -7$

Putting variables on left:

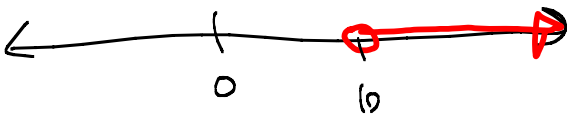
$$3x < 5x - 12$$
$$-5x \quad -5x$$
$$-2x < -12$$
$$\frac{-2x}{-2} > \frac{-12}{-2}$$ ← Reverse the inequality sign

$$\boxed{x > 6}$$

Putting variables on right side:

$$3x + 7 < 5x - 5$$
$$\quad +5 \quad\quad +5$$
$$3x + 12 < 5x$$
$$-3x \quad\quad -3x$$
$$12 < 2x$$
$$\frac{12}{2} < \frac{2x}{2}$$
$$6 < x$$

Rewrite as $\boxed{x > 6}$.



Ex:

$$\frac{2}{5}x + \frac{1}{3} \geq -\frac{5}{6}x + \frac{3}{4}$$

$$\frac{2}{5}x + \frac{5}{6}x \geq \frac{3}{4} - \frac{1}{3}$$

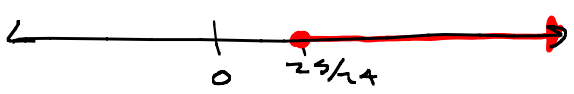
$$\left(\frac{6}{6}\right)\frac{2}{5}x + \left(\frac{5}{6}x\right)\left(\frac{5}{5}\right) \geq \frac{3}{4}\left(\frac{3}{3}\right) - \frac{1}{3}\left(\frac{4}{4}\right)$$

$$\frac{12}{30}x + \frac{25}{30}x \geq \frac{9}{12} - \frac{4}{12}$$

$$\frac{37}{30}x \geq \frac{5}{12}$$

$$\left(\frac{30}{37}\right)\left(\frac{37}{30}x\right) \geq \frac{5}{12}\left(\frac{30}{37}\right)$$

$$x \geq \frac{25}{74}$$



Combined Inequalities

Inequalities can be combined using OR or AND.

Example: Solve and graph.

$$2x + 3 < -4 \quad \text{OR} \quad 2x + 3 > 5$$

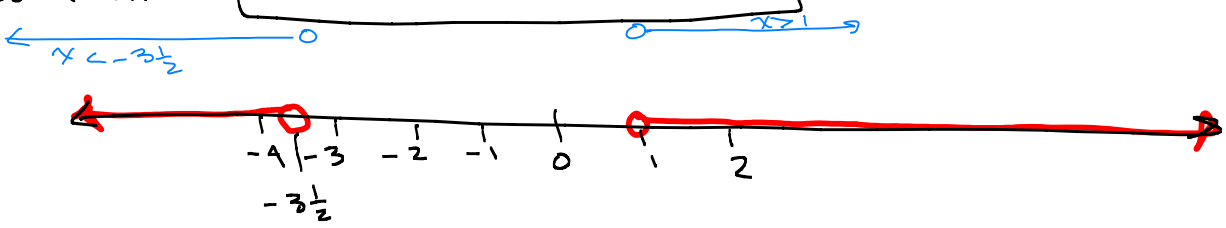
$$2x < -7 \qquad 2x > 2$$

$$\frac{2x}{2} < \frac{-7}{2} \qquad \frac{2x}{2} > 2$$

$$x < -\frac{7}{2} \qquad x > 1$$

$$x < -3\tfrac{1}{2}$$

Solution: $x < -3\frac{1}{2}$ OR $x > 1$



Ex: Solve $-6 \le 3x + 9 < 15$

This means: $-6 \le 3x + 9$ AND $3x + 9 < 15$

$-9 \quad -9$ $\qquad$ $-9 \quad -9$

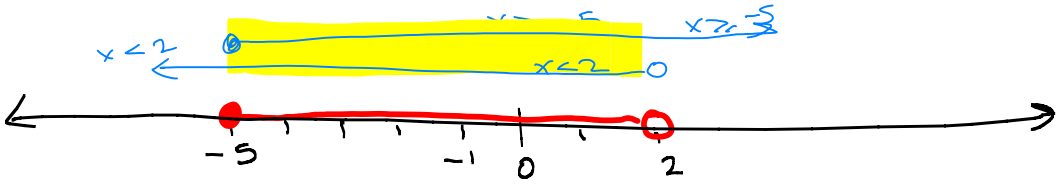
$-15 \le 3x$ $\qquad$ $3x < 6$

$\frac{-15}{3} \le \frac{3x}{3}$ $\qquad$ $\frac{3x}{3} < \frac{6}{3}$

$-5 \le x$ $\qquad$ $x < 2$

$x \ge -5$

AND



Put the pieces back into a combined inequality: $\boxed{-5 \le x < 2}$ ← final answer

Do this problem again: Keeping it together:

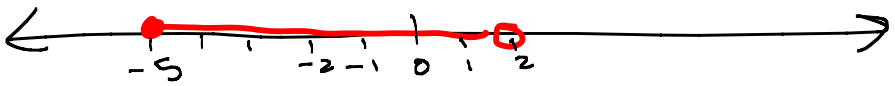
$-6 \le 3x + 9 < 15$ ← Subtract 9 from all 3 "sides"

$-9 \quad -9 \quad -9$

$-15 \le 3x < 6$

$\frac{-15}{3} \le \frac{3x}{3} < \frac{6}{3}$ ← Divide all 3 "sides" by 3

$\boxed{-5 \le x < 2}$



Ex: $83 \le 5 - x < 100$

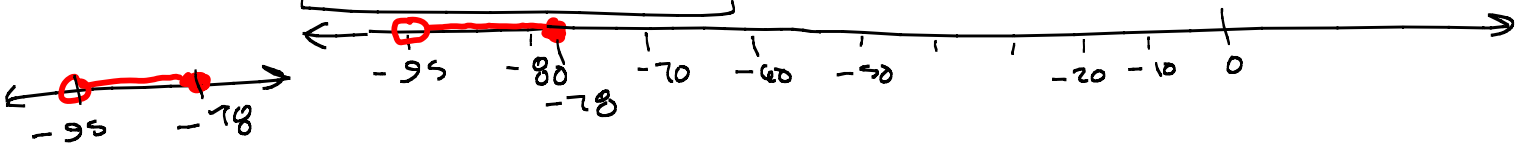
$-5 \quad -5 \quad -5$

$78 \le -x < 95$

$\frac{78}{-1} \ge \frac{-x}{-1} > \frac{95}{-1}$ ← Reverse all the inequality

$-78 \ge x > -95$

$\boxed{-95 < x \le -78}$



(83) $-2 \leq m-5 \leq 2$

$+5 \qquad +5 \qquad +5$

$3 \leq m \leq 7$

Note: Back of book has $3 \leq m \geqslant 7$.

This is wrong. When writing a combined inequality like this, both signs must point the same direction.

Homework Qs

2.6 #29

number of kids tickets: $x+6$

number of adult tickets: x

kids tix → compare to → # adult tix

$x+6$ x

$$\underbrace{4(x+6)}_{\text{total value of kids tix}} + \underbrace{6x}_{\text{total value of adult tix}} = 184$$