

3.5: Equations of Lines (continued)

2/17/2015

Note Title

Example: Find the equation of the line that passes through the points $(-1, -5)$ and $(2, 1)$.

$$\text{Find slope: } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-5)}{2 - (-1)} = \frac{1 + 5}{2 + 1} = \frac{6}{3} = 2$$

Once you have a point and a slope, there are 2 ways to get the equation of the line:

① Point-slope form: $y - y_1 = m(x - x_1)$

② Slope-intercept form: $y = mx + b$

Method A

① Point-slope form:

$$m = 2, x_1 = -1, y_1 = -5 \Rightarrow y - (-5) = 2(x - (-1))$$

(could choose either of the given points)

$$y + 5 = 2(x + 1)$$

$$y + 5 = 2x + 2$$

$$y = 2x - 3$$

Notice:

$y - y_1 = m(x - x_1)$ is a rearrangement of the slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Method B

② Slope-intercept form:

$$m = 2 \Rightarrow$$

$$y = mx + b$$

$$y = 2x + b$$

$$x = -1, y = -5 \Rightarrow$$

(could choose either of the given points)

$$-5 = 2(-1) + b$$

$$-5 = -2 + b$$

$$-3 = b$$

Put $b = -3$ into $y = 2x + b$:

$$b = -3 \Rightarrow$$

$$y = 2x - 3$$

is equation of the line.

If I used the point $(2, 1)$ instead:

$$x = 2, y = 1 \Rightarrow$$

$$y = 2x + b$$

$$1 = 2(2) + b$$

$$1 = 4 + b$$

$$-3 = b$$

So equation of line is $y = 2x - 3$.

3.5 #32

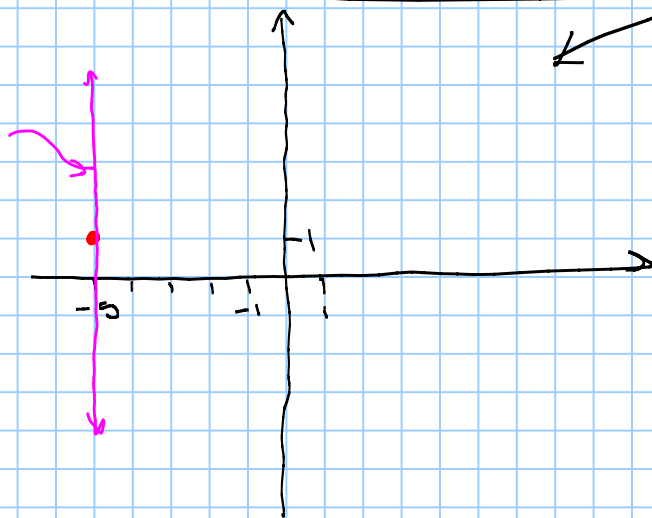
Find the equation of the line that passes through the point $(-5, 1)$ and has undefined slope.

Recall:

- * Horizontal lines have slopes of 0.
- * Vertical lines have undefined slope.

Sketch a graph:

eqn of this line is $x = -5$



The equation of the desired line is $x = -5$.

Parallel and perpendicular lines:

For diagonal lines:

- * Parallel lines have the same slope.

(slopes of parallel lines are equal.)

- * Perpendicular lines intersect at a 90° (right) angle.

Perpendicular lines have slopes that are opposite reciprocals.

(If one line has slope m , the other line has slope $-\frac{1}{m}$.)

Why?

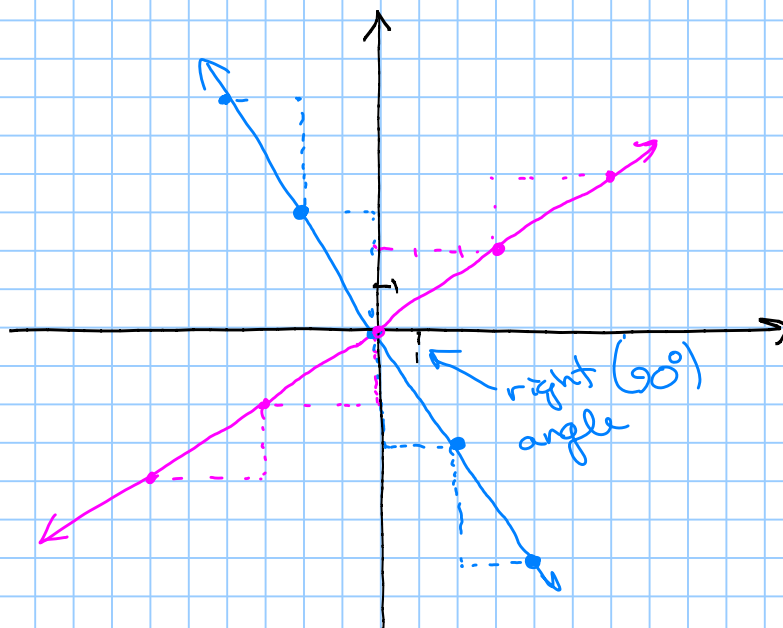
If a line has slope $\frac{2}{3}$, then

- 1) all lines parallel to it also have slope $\frac{2}{3}$.
- 2) all lines perpendicular to it have slope $-\frac{3}{2}$.

See next page for picture:

$$\text{Slope} = \frac{2}{3}$$

↑
"rise" = 2
"run" = 3



Perpendicular line
has slope $-\frac{3}{2}$

↙
"rise" = 3
"run" = 2

Example: Find the equation of the line that includes the point $(-1, 2)$ and is parallel to the line with equation $4x - 2y = 6$.

Find slope of the given line by writing in $y = mx + b$ form:

$$4x - 2y = 6$$

$$-2y = -4x + 6$$

$$\frac{-2y}{-2} = \frac{-4x}{-2} + \frac{6}{-2}$$

$$y = 2x - 3$$

So, slope of the given line is 2.

So, the slope of the desired line is also 2.
(because they are parallel)

From here, you can write the line using either point-slope form or slope-intercept form.

Method A

① Using point-slope form:

$$m = 2, x_1 = -1, y_1 = 2 \Rightarrow$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 2(x - (-1))$$

$$y - 2 = 2(x + 1)$$

$$y - 2 = 2x + 2$$

$$\boxed{y = 2x + 4}$$

Method B

②

Using slope-intercept form:

$$m = 2 \Rightarrow$$

$$x = -1, y = 2 \Rightarrow$$

$$y = mx + b$$

$$y = 2x + b$$

$$2 = 2(-1) + b$$

$$2 = -2 + b$$

$$4 = b$$

Need to find b.

Put $b = 4$ into $y = 2x + b$

$$\boxed{y = 2x + 4}$$

Ex: Find the equation of the line that passes through $(-4, 2)$ and is perpendicular to the line with equation $-3x - 5y = 7$.

Find slope of given line by writing in $y = mx + b$ form:

$$-3x - 5y = 7$$

$$-5y = 3x + 7$$

$$\frac{-5y}{-5} = \frac{3x}{-5} + \frac{7}{-5}$$

$$y = \underline{-\frac{3}{5}x} - \frac{7}{5}$$

So slope of given line is $-\frac{3}{5}$.

So slope of desired line is $+\frac{5}{3}$.
(because they're perpendicular)

Using $y = mx + b$ form:

$$y = mx + b$$

$$m = \frac{5}{3} \Rightarrow$$

$$y = \frac{5}{3}x + b$$

Must find b .

$$x = -4, y = 2 \Rightarrow$$

$$2 = \frac{5}{3}(-4) + b$$

$$2 = -\frac{20}{3} + b$$

$$\underline{\left(\frac{3}{3}\right)2} + \frac{20}{3} = b$$

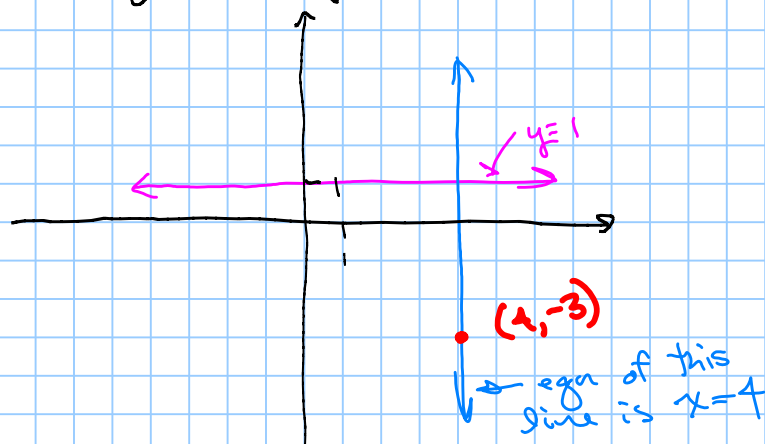
$$\frac{6}{3} + \frac{20}{3} = b$$

$$\underline{\frac{26}{3}} = b$$

Write the equation:

$$\boxed{y = \frac{5}{3}x + \frac{26}{3}}$$

Ex: Write the equation of the line that passes through the point $(4, -3)$ and is perpendicular to the line with equation $y = 1$.



$$y = 1$$

$$(0, 1)$$

$$(2, 1)$$

$$(-5, 1)$$

Equation is

$$\boxed{x = 4}$$