4.1 : Multiplication with Exponents

Note Title 2/24/2015

Recall: Exponents $\Rightarrow$ repeated multiplication

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 9 = \boxed{81}$$

Know these powers:

$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024$

$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$

$4^2 = 16$
$4^3 = 64$
$4^4 = 256$

$5^2 = 25$
$5^3 = 125$
$5^4 = 625$

$6^2 = 36$
$6^3 = 216$

$7^2 = 49$
$7^3 = 343$

$8^2 = 64$
$8^3 = 512$

$9^2 = 81$
$9^3 = 729$

$10^2 = 100$
$10^3 = 1000$
$10^4 = 10000$
$10^8 = 100\,000\,000$

Powers of positive and negative numbers

$$-3^2 = -(3)(3) = \boxed{-9}$$

$$(-3)^2 = (-3)(-3) = \boxed{9}$$

Ex: $-(3)^2 = -(9) = -9$

$-3^2 = -(3)^2 = -9$

Example: $(-3)^3 = (-3)(-3)(-3) = 9(-3) = \boxed{-27}$

$-3^3 = -(3)(3)(3) = \boxed{-27}$ same

$(-3)^6 = (-3)(-3)(-3)(-3)(-3)(-3) = \boxed{729}$

$(-3)^5 = (-3)(-3)(-3)(-3)(-3) = \boxed{-243}$

$-3^6 = -(3)^6 = \boxed{-729}$

$-3^5 = -(3)^5 = \boxed{-243}$ same

Important Fact:

* If you raise a negative number to an even power, the result is positive.

* If you raise a negative number to an odd power, the result is negative

Ex: $(-6)^3 = \boxed{-216}$

$(-2)^6 = \boxed{64}$

Ex: $-(-2)^4 = \boxed{-16}$

$-(-2) = \boxed{2}$

Note: $-(-2)^4 = -1(-2)^4 = -1(16) = \boxed{-16}$

$-3^2 = \boxed{-9}$

$-(-1)^{11} = -(-1) = \boxed{1}$

Properties (Laws) of Exponents

1) $x^a x^b = x^{a+b}$

Ex: $x^2 x^3 = (xx)(xxx) = x^5$

2) $(x^a)^b = x^{ab}$

Ex: $(x^2)^3 = (x^2)(x^2)(x^2)$
$= (xx)(xx)(xx) = x^6$

3) $\frac{x^a}{x^b} = x^{a-b}$

Ex: $\frac{x^6}{x^2} = \frac{\cancel{x}\cancel{x}xxxx}{\cancel{x}\cancel{x}} = \frac{x^4}{1} = x^4$

Note: $\frac{x^2}{x^6} = \frac{\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}xxxx} = \frac{1}{x^4}$

same as $\frac{x^2}{x^6} = x^{2-6} = x^{-4}$

} must be equal

So, $\frac{1}{x^4} = x^{-4}$ (Topic of Section 4.2)

4) $(xy)^a = x^a y^a$

Ex: $(xy)^3 = (xy)(xy)(xy) = x^3 y^3$

5) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

Ex: $\left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{x^3}{y^3}$

Ex: $2^2 \cdot 2^4 = 2^6 = \boxed{64}$

Ex: $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \boxed{\frac{16}{81}}$ ← Property #5

Ex: $\left(-\frac{4}{5}\right)^4 = \frac{4^4}{5^4} = \boxed{\frac{256}{625}}$

Ex: $x^3 x^8 = \boxed{x^{11}}$

Ex: $\frac{4^{11}}{4^9} = 4^{11-9} = 4^2 = \boxed{16}$

Ex: $\frac{2^5}{3^4} = \boxed{\frac{32}{81}}$

Ex: $\left(-\frac{3}{2}\right)^5 = -\frac{3^5}{2^5} = \boxed{-\frac{243}{32}}$

Ex: $2x^3x^4 = \boxed{2x^7}$

Ex: $(5a^4)(-2a) = (5a^4)(-2a^1) = \boxed{-10a^5}$

Ex: $(x^3y^5)^3 = (x^3)^3(y^5)^3$ ← Property #4

$= \boxed{x^9y^{15}}$ ← Property #2

Ex: $\left(\frac{y^3}{x}\right)^2 = \boxed{\frac{y^6}{x^2}}$ ← Properties #5 and #2

Ex: $(-2x^3)^5 = (-2)^5x^{15} = \boxed{-32x^{15}}$

Ex: $(-3x^2)^3(-2x^4)^4 = (-3)^3x^6(-2)^4x^{16}$

$= -27x^6(16)x^{16} =$

$= \boxed{-432x^{22}}$

Ex: $(-2x^2y)^4(-xy^3)^2 = (-2x^2y)^4(-1xy^3)^2$

$= (-2)^4x^8y^4(-1)^2x^2y^6$

$= 16x^8y^4(1)x^2y^6$

$= \boxed{16x^{10}y^{10}}$

$\frac{27}{16}$

$16\overline{)2}$

$\frac{27}{4}\overline{)32}$

In 4.1, skip the scientific notation (for now) until we get to the end of 4.2

4.2: Division with Exponents and Negative Exponents

Definition of a negative exponents.

$$x^{-n} = \frac{1}{x^n}$$

Note: $x^{-n} = \frac{x^{-n}}{1} = \frac{1}{x^n}$

$$\frac{1}{x^{-n}} = \frac{x^n}{1} = x^n$$

$$x^{-1} = \frac{x^{-1}}{1} = \frac{1}{x^1} = \frac{1}{x}$$

Ex: $5^{-2} = \frac{5^{-2}}{1} = \frac{1}{5^2} = \boxed{\frac{1}{25}}$

$$(-3)^{-4} = \frac{(-3)^{-4}}{1} = \frac{1}{(-3)^4} = \boxed{\frac{1}{81}}$$