Homework Qs

Note Title 3/5/2015

4.3 #75 $x^2\left(1-\frac{1}{x}-\frac{6}{x^2}\right)$

$$= x^2(1) + \frac{x^2}{1}\left(-\frac{1}{x}\right) + \frac{x^2}{1}\left(-\frac{6}{x^2}\right)$$

$$= x^2 - \frac{x^2}{x} - \frac{6x^2}{x^2}$$

$$= x^2 - \frac{x}{1} - \frac{6}{1} = \boxed{x^2 - x - 6}$$

Ex: $\frac{2}{3}x^2\left(\frac{1}{5}x^4 - \frac{3}{7}x^3 + \frac{1}{10}x\right)$

$$= \frac{2}{3}x^2\left(\frac{1}{5}x^4\right) + \frac{2}{3}x^2\left(-\frac{3}{7}x^3\right) + \frac{2}{3}x^2\left(\frac{1}{10}x\right)$$

$$= \frac{2}{15}x^6 - \frac{\cancel{6}^{\,2}}{\cancel{21}_{\,7}}x^5 + \frac{\cancel{2}^{\,1}}{\cancel{30}_{\,15}}x^3$$

$$= \boxed{\frac{2}{15}x^6 - \frac{2}{7}x^5 + \frac{1}{15}x^3}$$

4.4 #31 $\left(\frac{5}{9}x^3 + \frac{1}{3}x^2 - 2x + 1\right) - \left(\frac{2}{3}x^3 + x^2 + \frac{1}{2}x - \frac{3}{4}\right)$

$$= \frac{5}{9}x^3 + \frac{1}{3}x^2 - 2x + 1 - \frac{2}{3}x^3 - x^2 - \frac{1}{2}x + \frac{3}{4}$$

$$= \frac{5}{9}x^3 - \frac{2}{3}x^3 + \frac{1}{3}x^2 - x^2 - 2x - \frac{1}{2}x + 1 + \frac{3}{4}$$

$$= \frac{5}{9}x^3 - \frac{2}{3}x^3\left(\frac{3}{3}\right) + \frac{1}{3}x^2 - \frac{x^2}{1}\left(\frac{3}{3}\right) - \frac{2x}{1}\left(\frac{2}{2}\right) - \frac{1}{2}x + \frac{1}{1}\left(\frac{4}{4}\right) + \frac{3}{4}$$

$$= \frac{5}{9}x^3 - \frac{6}{9}x^3 + \frac{1}{3}x^2 - \frac{3}{3}x^2 - \frac{4}{2}x - \frac{1}{2}x + \frac{4}{4} + \frac{3}{4}$$

$$= \boxed{-\frac{1}{9}x^3 - \frac{2}{3}x^2 - \frac{5}{2}x + \frac{7}{4}}$$

4.4 #27 $\left(\frac{2}{3}x^2 - \frac{1}{5}x - \frac{3}{4}\right) + \left(\frac{4}{3}x^2 - \frac{4}{5}x + \frac{7}{4}\right)$

$$= \frac{2}{3}x^2 + \frac{4}{3}x^2 - \frac{1}{5}x - \frac{4}{5}x - \frac{3}{4} + \frac{7}{4}$$

$$= \frac{6}{3}x^2 - \frac{5}{5}x + \frac{4}{4}$$

$$= \boxed{2x^2 - x + 1}$$

4.6: Special Products

Recall:

Example: Multiply. $(3x^2 - x - 7)(-5x^2 + 2x - 6)$

$$3x^2(-5x^2 + 2x - 6) - x(-5x^2 + 2x - 6) - 7(-5x^2 + 2x - 6)$$

$$\begin{aligned} = -15x^4 + 6x^3 - 18x^2 & \\ + 5x^3 - 2x^2 + 6x & \\ + 35x^2 - 14x + 42 & \end{aligned}$$

$$= \boxed{-15x^4 + 11x^3 + 15x^2 - 8x + 42}$$

The "FOIL" method (for multiplying 2 binomials)

F: First

O: Outer

I: Inner

L: Last

Ex: $(2x - 3)(4x + 7)$

First Last Inner Outer

$$= 8x^2 + 14x - 12x - 21$$

$$= \boxed{8x^2 + 2x - 21}$$

Using distributive property:

$$(2x - 3)(4x + 7)$$

$$= 2x(4x + 7) - 3(4x + 7)$$

$$= 8x^2 + 14x - 12x - 21$$

$$= \boxed{8x^2 + 2x - 21}$$

4.5 #52 $(x+3)(x+4)(x+5)$

$= (x^2+4x+3x+12)(x+5)$

$= (x^2+7x+12)(x+5)$

$= x^2(x+5)+7x(x+5)+12(x+5)$

$= x^3+5x^2+7x^2+35x+12x+60$

$= \boxed{x^3+12x^2+47x+60}$

Perfect Squares

Example: $(x+8)^2$

$= (x+8)(x+8)$

$= x^2+8x+8x+64$

$= \boxed{x^2+16x+64}$

Perfect Square Pattern

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

Check:

$(a-b)^2$

$= (a-b)(a-b)$

$= a^2-ab-ba+b^2$

$= a^2-ab-ab+b^2$

$= a^2-2ab+b^2$ ✓ok

Ex: $(x-4)^2$

$= \boxed{x^2-8x+16}$

Note:

$(x-4)(x-4)$

$= x^2-4x-4x+16$

$= x^2-8x+16$

Ex: $(3x+5)^2$

$9x^2+30x+25$

OR $(3x+5)(3x+5)$

$= 9x^2+15x+15x+25$

$= 9x^2+30x+25$

Ex: $(2x-3)^2$

$\boxed{4x^2 - 12x + 9}$

Ex: $(x-5)(x+5)$

$= x^2 + \cancel{5x} - \cancel{5x} - 25$

$= \boxed{x^2 - 25}$

Difference of 2 squares Pattern

$$(a+b)(a-b) = a^2 - b^2$$

Check it:

$(a+b)(a-b)$

$= a^2 - \cancel{ab} + \cancel{ab} - b^2$

$= a^2 - b^2$ ✓ok

Ex: $(x+6)(x-6) = x^2 - 36$

Ex: $(x-9)(x+9) = x^2 - 81$

Ex: $(2x+5)(2x-5) = 4x^2 - 25$

work it out: $4x^2 - \cancel{10x} + \cancel{10x} - 25 = \boxed{4x^2 - 25}$

4.7: Dividing a polynomial by a monomial

Ex: Divide.

$$\frac{6x^4 - 8x^3 + 12x^2 - 9}{2x^2}$$

Break into separate fraction:

$$\frac{\overset{3}{\cancel{6}}x^4}{\underset{1}{\cancel{2}}x^2} - \frac{\overset{4}{\cancel{8}}x^3}{\underset{1}{\cancel{2}}x^2} + \frac{\overset{6}{\cancel{12}}x^2}{\underset{1}{\cancel{2}}x^2} - \frac{9}{2x^2}$$

$$= \frac{3x^2}{1} - \frac{4x}{1} + \frac{6}{1} - \frac{9}{2x^2} = \boxed{3x^2 - 4x + 6 - \frac{9}{2x^2}}$$

Note:

$\frac{2}{7} + \frac{3}{7}$

$= \frac{2+3}{7}$

$= \frac{5}{7}$

Ex: $$\frac{2x^4y^3 - 8x^3y + 7xy}{-2x^2y^2}$$

$$= \frac{\cancel{2}^{1}x^4y^3}{-\cancel{2}_{1}x^2y^2} - \frac{\cancel{8}^{4}x^3y}{-\cancel{2}_{1}x^2y^2} + \frac{7xy}{-2x^2y^2}$$

$$= -\frac{1\,x^2y}{1} + \frac{4\,x}{1\,y} - \frac{7}{2xy}$$

$$= \boxed{-x^2y + \frac{4x}{y} - \frac{7}{2xy}}$$

4.8: Dividing a Polynomial by a Polynomial
(where divisor has 2 or more terms)

Polynomial Long Division

Example: Divide $\frac{379}{12}$.

$$\begin{array}{r} 31 \\ 12\overline{)379} \\ -36 \\ \hline 19 \\ 12 \\ \hline 7 \end{array}$$

Write answer:

$$\frac{379}{12} = 31 + \frac{7}{12} \quad \left(\text{Usually written } 31\tfrac{7}{12}\right)$$

Check: $31(12) + 7$
$= 372 + 7 = 379$ ✓

$$\begin{array}{r} 31 \\ \times\ 12 \\ \hline 62 \\ 31 \\ \hline 372 \end{array}$$

Review of Terminology

$$\begin{array}{r} \text{Quotient} \\ \text{Divisor}\ \overline{)\ \text{Dividend}} \\ \sim\sim\sim \\ \sim\sim \\ \sim\sim \\ \sim \\ \hline \text{Remainder} \end{array}$$

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\text{Dividend} = (\text{Quotient})(\text{Divisor}) + \text{Remainder}$$

Ex: Divide.

$$\frac{3x^2+7x+9}{x+2}$$

$\frac{3x^2}{x}$ $\quad \frac{x}{x}$

$$\begin{array}{r} 3x + 1 \\ x+2 \overline{)\, 3x^2 + 7x + 9} \\ -(3x^2+6x) \\ \hline 0 + x + 9 \\ -(x+2) \\ \hline 7 \end{array}$$

$3x(x+2) = 3x^2+6x$

$1(x+2) = 1x+2$

Note:

$$\frac{3x^2+7x+9}{x+2} = \frac{3x^2}{x+2} + \frac{7x}{x+2} + \frac{9}{x+2}$$

but they don't reduce

Must do long division

Write answer: $\frac{3x^2+7x+9}{x+2} = \boxed{3x+1+\frac{7}{x+2}}$ answer

Check your answer:

$(3x+1)(x+2)+7$

$= 3x^2+6x+x+2+7 = 3x^2+7x+9$ ✓

Note: If you replace $x=10$ in this problem, you get

$$\frac{379}{12} = 31+\frac{7}{12}$$

Ex: Divide $\dfrac{x^3-6x^2+x+14}{x-5}$

$$\frac{x^3}{x} \qquad \frac{-x^2}{x} \qquad \frac{-4x}{x}$$

$$\begin{array}{rl}
 & x^2 \quad -x \quad -4 \\ \hline
x-5 \,\big) & x^3-6x^2+x+14 \\
 & -(x^3-5x^2) \\ \hline
 & -x^2+x \\
 & -(-x^2+5x) \\ \hline
 & -4x+14 \\
 & -(-4x+20) \\ \hline
 & -6
\end{array}$$

$x^2(x-5) = x^3-5x^2$

$-x(x-5) = -x^2+5x$

Write answer: $\dfrac{x^3-6x^2+x+14}{x-5} = \boxed{x^2-x-4+\dfrac{-6}{x-5}}$

$= \boxed{x^2-x-4-\dfrac{6}{x-5}}$

both are correct

Check our answer by multiplying: $(x-5)(x^2-x-4)-6$

$= x^3 - x^2 - 4x$
$\quad -5x^2+5x+20-6$

$= x^3-6x^2+x+14$ ✓

Important: When doing polynomial long division, insert 0's as placeholders for missing powers of x.

Example of this:

To divide $\dfrac{x^3+2x}{x-5}$, write: $x-5\overline{)\,x^3+0x^2+2x+0}$