Chapter 5: Factoring Polynomials

Note Title 3/24/2015

Ex: Write the number 18 in factored form.

$$18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3 \qquad 18 = \boxed{2 \cdot 3^2}$$

Ex: $(x+3)(x+4)$ ← factored form

$= x^2 + 7x + 12$ ← unfactored form

5.1: The Greatest Common Factor and Factoring by Grouping

To find the Greatest Common Factor (GCF):

* Find the largest number that divides into all the coefficients
* Find the highest power of each variable that is a factor of all the terms. (If a variable shows up in all the terms, choose the smallest power)
* Multiply the numerical part and the variable part to get the GCF.

Ex: Factor out the GCF.

$15x^4 - 6x^2$ GCF: $3x^2$

$= 3x^2\left(\frac{15x^4}{3x^2} - \frac{6x^2}{3x^2}\right)$

$= \boxed{3x^2(5x^2 - 2)}$

Check: $3x^2(5x^2 - 2)$

$= 15x^4 - 6x^2$ ✓ OK

Ex: Factor.

$48x^3 - 32x$

GCF: $8x$

Not the Greatest Common Factor

$= 8x(6x^2 - 4)$

can still factor out a 2

$= 8x(2)(3x^2 - 2)$

$= \boxed{16x(3x^2 - 2)}$

Check: $8x(6x^2 - 4)$

$= 48x^3 - 32x$ ok

Check: $16x(3x^2 - 2)$

$= 48x^3 - 32x$ ✓

	1
	16
	3
	48

Try again from the beginning:

$48x^3 - 32x$

GCF: $16x$

$\boxed{16x(3x^2 - 2)}$

Ex: Factor.

$24x^3 - 18x^2 + 6$

GCF: 6

$= \boxed{6(4x^3 - 3x^2 + 1)}$

Check: $6(4x^3 - 3x^2 + 1)$

$= 24x^3 - 18x^2 + 6$ ✓

Ex: $-30x^4y^3z^3 + 12y^6z^2 - 42x^4y^2z$

GCF: $6y^2z$

$= 6y^2z(-5x^4yz^2 + 2y^4z - 7x^4)$

Check it!

Factoring by Grouping

Example: Factor.

$$3x^3 - 7x^2 + 15x - 35$$
$$= (3x^3 - 7x^2) + (15x - 35)$$
$$= x^2(3x - 7) + 5(3x - 7)$$
$$= \boxed{(3x-7)(x^2+5)}$$

Check: $(3x-7)(x^2+5)$
$= 3x^3 + 15x - 7x^2 - 35$ ✓

Also correct: $\boxed{(x^2+5)(3x-7)}$

Ex: $4x^3 - 3x^2 - 36x + 27$
$= (4x^3 - 3x^2) + (-36x + 27)$
$= x^2(4x-3) - 9(4x-3)$
$= (4x-3)(x^2-9)$
$= \boxed{(4x-3)(x+3)(x-3)}$

$(a+b)(a-b) = a^2 - b^2$

Note: $(x+3)(x-3)$
$= x^2 - 3x + 3x - 9$
$= x^2 - 9$ "Difference of 2 Squares Pattern"
(we'll get to this in Section 5.4)

Ex: $2a^2b - 16a^2 + b - 8$
$(2a^2b - 16a^2) + (b - 8)$
$= 2a^2(b-8) + 1(b-8)$
$= \boxed{(b-8)(2a^2+1)}$

Check: $(b-8)(2a^2+1)$
$2a^2b + b - 16a^2 - 8$ ✓ ok

Ex: $-16x^2y - 4x^2 + 24xy + 6x$

$= 2x(-8xy - 2x + 12y + 3)$

$= 2x\left[(-8xy - 2x) + (12y + 3)\right]$

$= 2x\left[-2x(4y + 1) + 3(4y + 1)\right]$

$= 2x\left[(4y+1)(-2x+3)\right]$

$= \boxed{2x(4y+1)(-2x+3)}$

Ex: $7xy^2 - 20x + 2y^2 - 70x^2$

$= (7xy^2 - 20x) + (2y^2 - 70x^2)$

$= x(7y^2 - 20) + 2(y^2 - 35x^2)$

Parentheses don't match!
switch the order of the terms
(I will switch the 2nd & 3rd terms)

$7xy^2 + 2y^2 - 20x - 70x^2$

$= (7xy^2 + 2y^2) + (-20x - 70x^2)$

$= y^2(7x+2) - 10x(2 + 7x)$

$= \boxed{(7x+2)(y^2 - 10x)}$

Check: $(7x+2)(y^2-10x)$

$= 7xy^2 - 70x^2 + 2y^2 - 20x$ ✓

5.2: Factoring Trinomials (when the leading coefficient is 1)

(+) signs are the same
I want a sum of 7x for middle term

Ex: $x^2 + 7x + 10$

$(x + 2)(x + 5)$ ← final answer

Check: $x^2 + 5x + 2x + 10$
$= x^2 + 7x + 10$ ✓

(−) signs are opposite
want a difference of 10

Ex: $x^2 + 10x - 24$

$(x - 2)(x + 12)$

Check: $x^2 + 12x - 2x - 24$
$= x^2 + 10x - 24$ ✓

Also correct: $(x+12)(x-2)$

24
1 · 24
2 · 12
3 · 8
4 · 6

(+) signs are the same
want a sum of 10

Ex: $x^2 - 10x + 24$

$(x - 4)(x - 6)$

Check: $x^2 - 6x - 4x + 24$
$= x^2 - 10x + 24$ ✓

24
1 · 24
2 · 12
3 · 8
4 · 6

(−) signs are opposite
want a difference of 10

Ex: $x^2 - 10x - 24$

$(x + 2)(x - 12)$

Check: $x^2 - 12x + 2x - 24$
$x^2 - 10x - 24$

24
1 · 24
2 · 12
3 · 8
4 · 6

(+) signs are the same
want a sum of 10

Ex: $x^2 + 10x + 24$

$(x+4)(x+6)$

Check: $x^2 + 6x + 4x + 24$
$x^2 + 10x + 24$ ✓

24
1 · 24
2 · 12
3 · 8
4 · 6

Ex: $x^2 - 11xy - 12y^2$

(−) signs are opposite

want a difference of 11

$(x + 1y)(x - 12y)$

12

1 · 12

2 · 6

3 · 4

Check: $x^2 - 12xy + 1xy - 12y^2$

$x^2 - 11xy - 12y^2$

$\boxed{(x+y)(x-12y)}$

Ex: $-x^2 + 14x - 40$

Want leading term to be positive, so factor out -1

$-1(x^2 - 14x + 40)$

(+) signs the same

sum of 14

$\boxed{-(x-4)(x-10)}$

40

1 · 40

2 · 20

4 · 10

5 · 8

Check: $(x-4)(x-10)$

$= x^2 - 10x - 4x + 40$

$= x^2 - 14x + 40$ ✓ok

(we're checking it against $x^2 - 14x + 40$, not against the original)