# 5.3: Factoring Trinomials (with a leading coefficient $\neq 1$)

Note Title 3/26/2015

**Ex:** Factor.

$x^2 + 16x - 36$

(−) signs are opposite want a difference of 16

$(x - 2)(x + 18)$

Check: $x^2 + 18x - 2x - 36$
$= x^2 + 16x - 36$ ✓

36
1·36
2·18
3·12
4·9
6·6

**Ex:** $2x^2 - 18x + 28$

$= 2(x^2 - 9x + 14)$

(+) same signs want a sum of 9x

$= 2(x-2)(x-7)$

Check: $(x-2)(x-7)$
$= x^2 - 7x - 2x + 14$
$= x^2 - 9x + 14$ ✓

14
1·14
2·7

**Ex:** $-x^2 + 3x + 54$

$= -1(x^2 - 3x - 54)$

(−) signs are opposite want a difference of 3

$= -(x + 6)(x - 9)$

Check: $(x+6)(x-9)$
$= x^2 - 9x + 6x - 54$
$= x^2 - 3x - 54$

54
1·54
2·27
3·18
6·9

**Ex:** Factor.

$x^2 + 3x - 180$

(−) signs are opposite want a difference of 3

$(x - 12)(x + 15)$

Check: $x^2 + 15x - 12x - 180$
$= x^2 + 3x - 180$ ✓

12
15
60
12
180

3·60
12·15

1·180
2·90
45·4
18·10
6·30
9·20
5·36

180
2·90
2·45·2
2·9·5·2
2·3·3·5·2

Ex: $x^2 + 7x - 12$ ← (-) signs are opposite want a difference of 7

12
1 · 12
2 · 6
3 · 4

Try: $(x+3)(x-4)$

Check $x^2 - 4x + 3x - 12$
$= x^2 - x - 12$ No!

This polynomial does not factor.
(This polynomial is **prime**.)

**Prime number:** A natural number that can only be factored (using natural number factors) as a product of 1 and itself.

**Prime polynomial:** cannot can only be factored (using integer coefficients) except by factoring out $\pm 1$.

## 5.3: Factoring Trinomials (when the leading coefficient > 1)

Ex: Factor.

$2x^2 + 9x + 10$ ← (+) signs are the same want a sum of 9x for middle term

10
1 · 10
2 · 5

Try $(2x+1)(x+10)$

Check: $2x^2 + 20x + 1x + 10$
$= 2x^2 + 21x + 10$ No!

Try $(2x+10)(x+1)$

Check: $2x^2 + 2x + 10x + 10$
$= 2x^2 + 12x + 10$ No!

Try $(2x+2)(x+5)$

Check: $2x^2 + 10x + 2x + 10$
$= 2x^2 + 12x + 10$ No!

Try $\boxed{(2x+5)(x+2)}$ ← correct answer

Check: $2x^2 + 4x + 5x + 10 = 2x^2 + 9x + 10$ ✓ok

Ex: $2x^2 - 25x + 12$ (+) same signs

$= (2x - 1)(x - 12)$

sum of 25x for middle term

Check: $2x^2 - 24x - 1x + 12$

$= 2x^2 - 25x + 12$ ✓

$2x^2$ ∧ $2x \cdot x$ — $-1x$ — $-24x$ — $12$ ∧ $-1 \cdot -12$, $-2 \cdot -6$, $-3 \cdot -4$

Ex: $4x^2 - 7x - 15$ (−) signs are opposite

$(4x + 5)(x - 3)$

want a difference of 7x for middle term

Check: $4x^2 - 12x + 5x - 15$

$4x^2 - 7x - 15$ ✓ ok

$4x^2$ ∧ $2x \cdot 2x$, $4x \cdot x$ — $5x$, $12x$ — $15$ ∧ $1 \cdot 15$, $3 \cdot 5$

Ex: $6x^2 + 19x + 14$ (+) same signs

$(6x + 7)(x + 2)$

want a sum of 19x for middle term

Check:

$6x^2 + 12x + 7x + 14$

$= 6x^2 + 19x + 14$ ✓

Also correct: $(x+2)(6x+7)$

$6x^2$ ∧ $6x \cdot x$, $2x \cdot 3x$ — $12x$, $7x$ — $14$ ∧ $1 \cdot 14$, $2 \cdot 7$

## 5.4: Difference of Two Squares and Perfect Square Trinomials

**Difference of 2 Squares Factorization**

$$a^2 - b^2 = (a+b)(a-b)$$

Check: $(a+b)(a-b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$ ✓

**Ex:** Factor.

$x^2 - 49$

$\boxed{(x+7)(x-7)}$

Check: $x^2 - 7x + 7x - 49$

$= x^2 - 49$ ✓ ok

**Ex:** $4x^2 - 25y^2$

$= (2x)^2 - (5y)^2$

$= \boxed{(2x-5y)(2x+5y)}$

**Ex:** $d^2 - 1$

$\boxed{(d-1)(d+1)}$

Check: $d^2 + 1d - 1d - 1$

$= d^2 - 1$

**Ex:** $2x^3 - 50x$

$= 2x(x^2 - 25)$

$= \boxed{2x(x-5)(x+5)}$

**Perfect Square Trinamials**

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

**Ex:** $x^2 - 12x + 36$

(+) same signs want sum of 12x

$(x-6)(x-6)$

Check: $x^2 - 6x - 6x + 36$

$x^2 - 12x + 36$ ✓ ok

$\boxed{(x-6)^2}$ ← final answer