

Review of Factoring so far

Note Title

3/31/2015

Ex: Factor

$$32x^3 - 50xy^2$$

$$2x(16x^2 - 25y^2)$$

$$= 2x((4x)^2 - (5y)^2)$$

$$= \boxed{2x(4x+5y)(4x-5y)}$$

GCF: $2x$

Recall: Difference of 2 squares factorization

$$a^2 - b^2 = (a+b)(a-b)$$

$$\text{Check: } (4x+5y)(4x-5y)$$

$$= 16x^2 + 20xy - 20xy - 25y^2$$

$$= 16x^2 - 25y^2 \checkmark$$

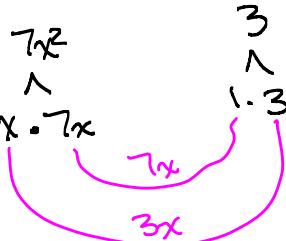
Ex: $-7x^2 + 10x - 3$

$$= -1(7x^2 - 10x + 3)$$

$$= \boxed{-(x-1)(7x-3)}$$

(+) same signs want a sum of $10x$ for middle term

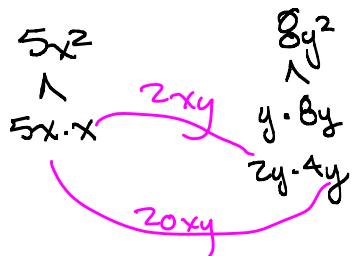
$$= \boxed{-(7x-3)(x-1)}$$



$$\text{Ex: } 5x^2 - 22xy + 8y^2$$

$$\boxed{(5x-2y)(x-4y)}$$

(+) same signs want sum of $22xy$ for middle term



$$\text{Check: } 5x^2 - 20xy - 2xy + 8y^2$$

$$= 5x^2 - 22xy + 8y^2 \checkmark$$

Ex: $3x^2 + 18x + 27$

$$= 3(x^2 + 6x + 9)$$

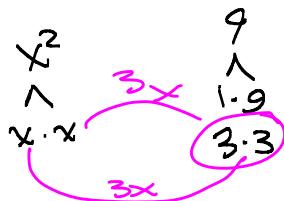
$$= 3(x+3)(x+3)$$

$$\text{Check: } (x+3)(x+3)$$

$$= x^2 + 3x + 3x + 9$$

$$= x^2 + 6x + 9 \checkmark$$

Answer: $\boxed{3(x+3)^2}$



Ex. $48x^2 + 38x - 21$

$$(6x + 7)(8x - 3)$$

(→)
 signs are opposite
 want difference of $38x$
 for middle term

Check: $48x^2 - 18x + 56x - 21$
 $= 48x^2 + 38x - 21 \checkmark$

$$\begin{array}{r}
 48x^2 \\
 \times 1 \\
 \hline
 48x^2
 \end{array}$$

$$\begin{array}{r}
 21 \\
 \times 1 \\
 \hline
 21
 \end{array}$$

$$\begin{array}{r}
 18x \\
 + 56x \\
 \hline
 38
 \end{array}
 \checkmark$$

Ex. Factor.

$$16x^2 - 56xy + 49y^2$$

Try $(4x - 7y)^2$

Check: $(4x - 7y)(4x - 7y)$
 $= 16x^2 - 28xy - 28xy + 49y^2$
 $= 16x^2 - 56xy + 49y^2 \checkmark$

$$\begin{array}{r}
 128 \\
 + 28 \\
 \hline
 56
 \end{array}$$

Answer: $(4x - 7y)^2$

Ex. $4x^2 - 20x + 9$

Try $(2x - 3)^2$

Check: $(2x - 3)(2x - 3)$
 $= 4x^2 - 6x - 6x + 9$
 $= 4x^2 - 12x + 9 \text{ No!!}$

Start over: $4x^2 - 20x + 9$

$$(2x - 1)(2x - 9)$$

(↑) same signs
 sum of $20x$

Check:

$$\begin{array}{r}
 4x^2 - 18x - 2x + 9 \\
 4x^2 - 20x + 9 \checkmark
 \end{array}$$

$$\begin{array}{r}
 4x^2 \\
 \times 1 \\
 \hline
 4x^2
 \end{array}$$

$$\begin{array}{r}
 9 \\
 \times 1 \\
 \hline
 9
 \end{array}$$

$$\begin{array}{r}
 18x \\
 + 2x \\
 \hline
 20x
 \end{array}$$

Ex: Factor.

$$x^4 - 13x^2 + 36$$

$$(x^2 - 9)(x^2 - 4)$$

$$(x+3)(x-3)(x+2)(x-2)$$

(+) same signs
sum of $13x^2$ for middle term

Check:

$$(x^2 - 9)(x^2 - 4)$$

$$= x^4 - 4x^2 - 9x^2 + 36$$

$$= x^4 - 13x^2 + 36 \quad \checkmark$$

36
1
1.36
2.18
3.12
4.9
6.6

Recall: $x^2 + \text{positive}$
is always prime
(cannot be factored)

5.5: Sum and Difference of Two Cubes

Know these formulas:

Sum and Difference of 2 Cubes Factorization

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Mnemonic Device: (for remembering the \pm signs)

M Match

S Same

O Opposite

O Opposite

P Positive

A Always

P Positive

Perfect Cubes

$$1^3 = 1$$

$$6^3 = 216$$

$$\frac{6}{4} \cdot \frac{9}{1}$$

$$2^3 = 8$$

$$7^3 = 343$$

$$\frac{8}{4} \cdot \frac{9}{1}$$

$$3^3 = 27$$

$$8^3 = 512$$

$$\frac{8}{4} \cdot \frac{9}{1}$$

$$4^3 = 64$$

$$9^3 = 729$$

$$\frac{8}{4} \cdot \frac{9}{1}$$

$$5^3 = 125$$

$$10^3 = 1000$$

$$\frac{8}{4} \cdot \frac{9}{1}$$

Ex: Factor.

$$\begin{aligned}
 & y^3 - 27 \\
 &= y^3 - 3^3 \\
 &= (y - 3)(y^2 + 3y + 9) \\
 &= \boxed{(y-3)(y^2+3y+9)}
 \end{aligned}$$

use
 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
 with
 $a = y, b = 3$

check: $(y-3)(y^2+3y+9)$

$$\begin{aligned}
 &= y^3 + 3y^2 + 9y \\
 &\quad - 3y^2 - 9y - 27 \\
 &= y^3 - 27 \checkmark
 \end{aligned}$$

Ex: Factor.

$$\begin{aligned}
 & 8x^3 + 125y^3 \\
 &= (2x)^3 + (5y)^3 \\
 &= (2x + 5y)((2x)^2 - 2x(5y) + (5y)^2) \\
 &= \boxed{(2x + 5y)(4x^2 - 10xy + 25y^2)}
 \end{aligned}$$

use

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Careful! Common mistake:

$$(2x + 5y)(2x^2 - 10xy + 5y^2)$$

must square both
 the variable
 and the
 number

Ex:

Factor.

$$\begin{aligned}
 & 2x^4 + 5x^3 - 2x - 5 \\
 &= (2x^4 + 5x^3) + (-2x - 5) \\
 &= x^3(2x + 5) - 1(2x + 5) \\
 &= (2x + 5)(x^3 - 1) \\
 &= (2x + 5)(x^3 - 1^3) \\
 &= (2x + 5)(x - 1)(x^2 + 1x + 1^2) \\
 &= \boxed{(2x + 5)(x - 1)(x^2 + x + 1)}
 \end{aligned}$$

5.6: Factoring: A General Review

Work these problems on your own!
Extremely important!

5.7: Solving Quadratic Equations by Factoring

A quadratic equation (in x) is an equation that can be written in the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is standard form for a quadratic

↑ ↑ ↑
 quadratic term linear term constant term

Zero Product Property (Zero Product Theorem)

If the product of two numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB=0$, then $A=0$ or $B=0$.

Ex: Solve.

$$x^2 - 24 = 10x$$

-10x -10x

[wrote in standard form $ax^2 + bx + c = 0$]

(\rightarrow)
opposite
signs
want
difference
of 10x

$$x^2 - 10x - 24 = 0$$

$$(x+2)(x-12) = 0$$

[Factor the non-zero side]

$x+2=0$ OR $x-12=0$

-2 +2 -2 +2

$$x = -2 \quad x = 12$$

[set each factor equal to 0]
(Zero Product Property)

24
1
1·24
12·12
3·8
4·6

Solution Set: $\{-2, 12\}$

Check on next page

$$x^2 - 24 = 10x$$

Check:

$$\begin{array}{l} \underline{x = -2} \\ (-2)^2 - 24 = 10(-2) \\ 4 - 24 = -20 \\ -20 = -20 \checkmark \end{array}$$

$$x^2 - 24 = 10x$$

$$\begin{array}{l} \underline{x = 12} \\ 12^2 - 24 = 10(12) \\ 144 - 24 = 120 \\ 120 = 120 \checkmark \end{array}$$