Review of Factoring so Far

Note Title 3/31/2015

Ex: Factor

$32x^3 - 50xy^2$

GCF: $2x$

$2x(16x^2 - 25y^2)$

Recall: Difference of 2 squares factorization

$a^2 - b^2 = (a+b)(a-b)$

$= 2x((4x)^2 - (5y)^2)$

$= \boxed{2x(4x+5y)(4x-5y)}$

Check: $(4x+5y)(4x-5y)$

$= 16x^2 - 20\cancel{xy} + 20\cancel{xy} - 25y^2$

$= 16x^2 - 25y^2$ ✓

Ex: $-7x^2 + 10x - 3$

$= -1(7x^2 - 10x + 3)$

(+) same signs
want a sum of 10x for middle term

$= \boxed{-(x - 1)(7x - 3)}$

$= \boxed{-(7x - 3)(x - 1)}$

$7x^2$; $x \cdot 7x$

3 ; $1 \cdot 3$

$7x$

$3x$

Ex: $x^2 - 64$

$x^2 - 8^2$

$= \boxed{(x+8)(x-8)}$

Check:

$x^2 - 8x + 8x - 64$

$= x^2 - 64$ ✓

Check: $(x-1)(7x-3)$

$= 7x^2 - 3x - 7x + 3$

$= 7x^2 - 10x + 3$ ✓

Ex: $5x^2 - 22xy + 8y^2$

(+) same signs
want sum of 22xy
for middle term

$\boxed{(5x - 2y)(x - 4y)}$

$5x^2$; $5x \cdot x$

$8y^2$; $y \cdot 8y$; $2y \cdot 4y$

$2xy$

$20xy$

Check: $5x^2 - 20xy - 2xy + 8y^2$

$= 5x^2 - 22xy + 8y^2$ ✓

Ex: $3x^2 + 18x + 27$

(+) same signs
sum of 6x

$= 3(x^2 + 6x + 9)$

$= 3(x + 3)(x + 3)$

x^2 ; $x \cdot x$

9 ; $1 \cdot 9$; $3 \cdot 3$

$3x$

$3x$

Check: $(x+3)(x+3)$

$= x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$ ✓

Answer: $\boxed{3(x+3)^2}$

Ex: $48x^2 + 38x - 21$

(−) signs are opposite
want difference of $38x$
for middle term

$$\boxed{(6x + 7)(8x - 3)}$$

Check: $48x^2 - 18x + 56x - 21$

$= 48x^2 + 38x - 21$ ✓

$48x^2$: $x \cdot 48x$, $2x \cdot 24x$, $3x \cdot 16x$, $4x \cdot 12x$, $6x \cdot 8x$

21: $1 \cdot 21$, $3 \cdot 7$

$18x$ $56x$

$56 - 18 = 38$ ✓

Ex: Factor.

$16x^2 - 56xy + 49y^2$

Try $(4x - 7y)^2$

Check: $(4x - 7y)(4x - 7y)$

$= 16x^2 - 28xy - 28xy + 49y^2$

$= 16x^2 - 56xy + 49y^2$ ✓

Answer: $\boxed{(4x - 7y)^2}$

$28 + 28 = 56$

Ex: $4x^2 - 20x + 9$

Try $(2x - 3)^2$

Check: $(2x - 3)(2x - 3)$

$= 4x^2 - 6x - 6x + 9$

$= 4x^2 - 12x + 9$ No!!

Start over: $4x^2 - 20x + 9$

(+) same signs
sum of $20x$

$$\boxed{(2x - 1)(2x - 9)}$$

Check:

$4x^2 - 18x - 2x + 9$

$4x^2 - 20x + 9$ ✓

$4x^2$: $x \cdot 4x$, $2x \cdot 2x$

9: $1 \cdot 9$, $3 \cdot 3$

$2x$ $18x$

Ex: Factor.

$x^4 - 13x^2 + 36$

(+) same signs sum of $13x^2$ for middle term

$(x^2 - 9)(x^2 - 4)$

$\boxed{(x+3)(x-3)(x+2)(x-2)}$

Check:

$(x^2-9)(x^2-4)$

$= x^4 - 4x^2 - 9x^2 + 36$

$= x^4 - 13x^2 + 36$ ✓

36

1·36
2·18
3·12
4·9
6·6

Recall: x^2 + positive is always prime (cannot be factored)

5.5: Sum and Difference of Two Cubes

Know these formulas:

Sum and Difference of 2 Cubes Factorization

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Mnemonic Device: (for remembering the ± signs)

M	Match	S	Same
O	Opposite	O	Opposite
P	Positive	A	Always
		P	Positive

Perfect Cubes

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$

$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$

$$\begin{array}{r} 6 \\ 49 \\ \times 7 \\ \hline 343 \end{array}$$

$$\begin{array}{r} 81 \\ \times 9 \\ \hline 729 \end{array}$$

$$\begin{array}{r} 64 \\ \times 8 \\ \hline 512 \end{array}$$

Ex: Factor.

$$y^3 - 27$$
$$= y^3 - 3^3$$
$$= (y-3)(y^2 + 3y + 3^2)$$
$$= \boxed{(y-3)(y^2+3y+9)}$$

Use
$$a^3 - b^3$$
$$= (a-b)(a^2 + ab + b^2)$$
with
$a = y$, $b = 3$

Check: $(y-3)(y^2+3y+9)$
$$= y^3 + \cancel{3y^2} + \cancel{9y}$$
$$- \cancel{3y^2} - \cancel{9y} - 27$$
$$= y^3 - 27 \checkmark$$

Ex: Factor.

$$8x^3 + 125y^3$$
$$= (2x)^3 + (5y)^3$$
$$= (2x + 5y)((2x)^2 - 2x(5y) + (5y)^2)$$
$$= \boxed{(2x+5y)(4x^2 - 10xy + 25y^2)}$$

Use
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Careful! Common mistake:
$$(2x+5y)(2x^2 - 10xy + 5y^2)$$
must square both the variable and the number

Ex:
Factor.

$$2x^4 + 5x^3 - 2x - 5$$
$$= (2x^4 + 5x^3) + (-2x - 5)$$
$$= x^3(2x+5) - 1(2x+5)$$
$$= (2x+5)(x^3 - 1)$$
$$= (2x+5)(x^3 - 1^3)$$
$$= (2x+5)(x-1)(x^2 + 1x + 1^2)$$
$$= \boxed{(2x+5)(x-1)(x^2+x+1)}$$

5.6: Factoring: A General Review

Work these problems on your own!

Extremely Important!

5.7: Solving Quadratic Equations by Factoring

A quadratic equation (in x) is an equation that can be written in the form $ax^2+bx+c=0$, where a, b, c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is standard form for a quadratic

quadratic term — linear term — constant term

Zero Product Property (Zero Product Theorem)

If the product of two numbers is 0, then at least one of the numbers must be 0.

In other words, if $AB=0$, then $A=0$ or $B=0$.

Ex: Solve.

$$x^2 - 24 = 10x$$

$-10x \qquad -10x$ [write in standard form $ax^2+bx+c=0$]

(−): opposite signs want difference of 10x

$$x^2 - 10x - 24 = 0$$

$$(x+2)(x-12)=0$$ [Factor the non-zero side]

$x+2=0$ OR $x-12=0$ [Set each factor equal to 0] (Zero Product Property)

$-2 \quad -2 \qquad +12 \quad +12$

$x=-2 \qquad x=12$

24: 1·24, 2·12, 3·8, 4·6

$$\boxed{\text{Solution Set: } \{-2, 12\}}$$

Check on next page

$$x^2 - 24 = 10x$$

Check:

$x=-2$ | $(-2)^2 - 24 = 10(-2)$

$4 - 24 = -20$

$-20 = -20$ ✓

$$x^2 - 24 = 10x$$

$x=12$ | $12^2 - 24 = 10(12)$

$144 - 24 = 120$

$120 = 120$ ✓