5.7: Solving Quadratic Equations by Factoring (continued)

Steps for solving a quadratic equation:

1) Write in standard form: $ax^2+bx+c=0$.
2) Factor the nonzero side.
3) Set each factor equal to 0. (use Zero Product Property)
4) Solve the resulting linear equations.

Example: Solve.

$$x^2+60=19x$$

$$x^2-19x+60=0$$

(+) same signs want sum of 19x

$$(x-15)(x-4)=0$$

x^2
$x \cdot x$

60
1·60
2·30
3·20
4·15
5·12
6·10

15x
4x

$x-15=0$ OR $x-4=0$
$x=15$ | $x=4$

Solution Set: $\boxed{\{15,4\}}$

Check:
$(x-15)(x-4)$
$=x^2-4x-15x+60$
$=x^2-19x+60$ ✓

Check our answers: $x^2+60=19x$

$x=4$

$4^2+60=19(4)$
$16+60=76$
$76=76$ ✓ok

$x=15$

$15^2+60=19(15)$
$225+60=285$
$285=285$ ✓

3
19
4
76

4
19
×15
95
1
19
285

2
15
15
75
15
225

Ex: Solve.

$$16 = x^2$$
$$0 = x^2 - 16$$
$$x^2 - 16 = 0$$
$$(x+4)(x-4) = 0$$

$x + 4 = 0$ OR $x - 4 = 0$

$x = -4$ | $x = 4$

Solution Set: $\boxed{\{-4, 4\}}$ OR $\boxed{\{\pm 4\}}$

"plus or minus 4"

Check:

$(x+4)(x-4)$

$= x^2 - \cancel{4x} + \cancel{4x} - 16$

$= x^2 - 16$ ✓

Example: solve.

$$12x^2 + 76x + 24 = 0$$
$$2(6x^2 + 38x + 12) = 0$$
$$2 \cdot 2(3x^2 + 19x + 6) = 0$$
$$4(3x^2 + 19x + 6) = 0$$

(+) same signs

want sum of $19x$

$$4(x + 6)(3x + 1) = 0$$

$4 = 0$ Never true OR $x + 6 = 0$, $x = -6$ OR $3x + 1 = 0$

$3x = -1$

$\frac{3x}{3} = \frac{-1}{3}$

$x = -\frac{1}{3}$

Solution Set: $\boxed{\left\{-6, -\frac{1}{3}\right\}}$

$$\begin{array}{r} 38 \\ 2\overline{)76} \\ \underline{6} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

$3x^2$ | 6

$x \cdot 3x$ | $1 \cdot 6$, $2 \cdot 3$

$1x$

$18x$

$$\begin{array}{r} {}^{3}19 \\ \underline{4} \\ 76 \end{array}$$

Check:

$(x+6)(3x+1)$

$= 3x^2 + x + 18x + 6$

$= 3x^2 + 19x + 6$ ✓

$4(3x^2 + 19x + 6)$

$= 12x^2 + 76x + 24$ ✓

Ex: solve. $x^3 - 6x^2 = -9x$

$$x^3 - 6x^2 + 9x = 0$$
$$x(x^2 - 6x + 9) = 0$$
$$x(x-3)(x-3) = 0$$

$x = 0$ OR $x - 3 = 0$ OR $x - 3 = 0$

$x = 3$ | $x = 3$

Solution Set $\boxed{\{0, 3\}}$

Note: This is not a quadratic (degree = 2) equation.
This is a cubic equation (degree = 3)

Check: $(x-3)(x-3)$

$= x^2 - 3x - 3x + 9$

$= x^2 - 6x + 9$ ✓

Ex: $-14x^2 + 24x = -2x^3$

$2x^3 - 14x^2 + 24x = 0$

$2x(x^2 - 7x + 12) = 0$

$2x(x - 3)(x - 4) = 0$

$2x = 0$ or $x - 3 = 0$ or $x - 4 = 0$

$\frac{2x}{2} = \frac{0}{2}$ | $x = 3$ | $x = 4$

$x = 0$

Solution Set: $\boxed{\{0, 3, 4\}}$

12

$\wedge$

$1 \cdot 12$

$2 \cdot 6$

$3 \cdot 4$

Check:

$(x-3)(x-4)$

$= x^2 - 4x - 3x + 12$

$= x^2 - 7x + 12$ ✓

(-) opposite signs

want difference of $-7x$

Ex: $6x^2 - 7x - 5 = 0$

$(2x + 1)(3x - 5) = 0$

$2x + 1 = 0$ or $3x - 5 = 0$

$2x = -1$ | $3x = 5$

$\frac{2x}{2} = \frac{-1}{2}$ | $\frac{3x}{3} = \frac{5}{3}$

$x = -\frac{1}{2}$ | $x = \frac{5}{3}$

Solution Set: $\boxed{\left\{-\frac{1}{2}, \frac{5}{3}\right\}}$

$6x^2$ $\quad$ 5

$\wedge$ $\quad$ $\wedge$

$x \cdot 6x$ $\quad$ $1 \cdot 5$

$2x \cdot 3x$

$3x$

$10x$

Check:

$(2x+1)(3x-5)$

$6x^2 - 10x + 3x - 5$

$6x^2 - 7x - 5$ ✓

5.8: Applications of Quadratic Equations (Word Problems!)

Example: The length of a rectangle is 6" more than four times its width. The area of the rectangle is 70 square inches. Find the length and width.

$4x$

length: $4x + 6$

width: x

length $\xrightarrow[\text{to}]{\text{compared}}$ width

$4x + 6$ $\quad$ x

Cont'd next page

Previous example continued:

$$\text{Area of rectangle} = (\text{length})(\text{width})$$

$$70 = (4x+6)(x)$$

$$x(4x+6) = 70$$

$$4x^2+6x = 70$$

$$4x^2+6x-70=0$$

$$2(2x^2+3x-35)=0$$

$$2(2x-7)(x+5)=0$$

(-) signs are opposite. want difference of $3x$

$2x^2$ ∧ $2x \cdot x$

35 ∧ $1 \cdot 35$, $5 \cdot 7$

$7x$

$10x$

$2=0$ never true OR $2x-7=0$, $2x=7$, $x=\frac{7}{2}$ OR $x+5=0$, $x=-5$ Throw out. A negative number does not make sense for a dimension.

Check:

$$(2x-7)(x+5)$$
$$=2x^2+10x-7x-35$$
$$=2x^2+3x-35 \checkmark$$

width: $x = \frac{7}{2} = 3\frac{1}{2}$ inches

length: $4x+6 = \frac{4}{1}\left(\frac{7}{2}\right)+6$

$$= \frac{28}{2}+6$$
$$= 14+6 = 20 \text{ inches}$$

$\boxed{\text{The width is } 3\frac{1}{2} \text{ inches and the length is 20 inches.}}$

Check our answers:

1st sentence: 4 times width: $4\left(\frac{7}{2}\right) = \frac{4}{1}\left(\frac{7}{2}\right) = \frac{28}{2} = 14$

6" more: 20" ✓

2nd sentence: Area = 70 in^2?

$$(\text{width})(\text{length}) = \frac{7}{2}\text{in}\left(\frac{20\text{in}}{1}\right) = \frac{140}{2}\text{in}^2 = 70\text{ in}^2 \checkmark$$

Ex: The height of a triangle is 3 feet less than five times its base. The area of the triangle is 13 ft². Find the base and height.

base: x

height: $5x - 3$

height $\xrightarrow[\text{to}]{\text{compared}}$ base

x

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$

$$13 = \frac{1}{2}(x)(5x-3)$$

$$\frac{1}{2}x(5x-3) = 13$$

Multiply both sides by 2 to get rid of fractions.

$$\left(\frac{\cancel{2}}{1}\right)\frac{1}{\cancel{2}}x(5x-3) = 13(2)$$

$$x(5x-3) = 26$$

$$5x^2 - 3x = 26$$

$$5x^2 - 3x - 26 = 0$$

(−) opposite signs

difference of 3x

$$(5x - 13)(x + 2) = 0$$

$5x - 13 = 0$

$5x = 13$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{13}{5}$

$x = \frac{13}{5}$

$x + 2 = 0$

$x = -2$

Throw out!

Negative makes no sense for a dimension

$5x^2$ 26

$x \cdot 5x$ $1 \cdot 26$

$2 \cdot 13$

$10x$

$13x$

Check:

$(5x-13)(x+2)$

$= 5x^2 + 10x - 13x - 26$

$= 5x^2 - 3x - 26$ ✓

base: $x = \frac{13}{5}$ ft $= 2\frac{3}{5}$ ft

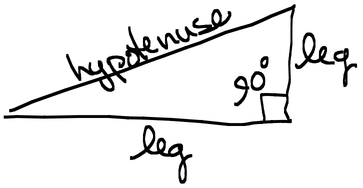
height: $5x - 3 = \frac{\cancel{5}}{1}\left(\frac{13}{\cancel{5}}\right) - 3 = 13 - 3 = 10$ ft

The base is $2\frac{3}{5}$ ft and the height is 10 ft.

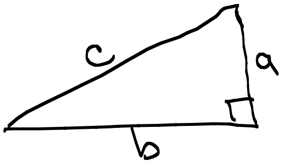
Ex.: The hypotenuse of a right triangle is 2" longer than the long leg. The long leg is 4" longer than twice the short leg. Find the lengths of all the sides.

Recall

Right Triangle:
(has 90° angle)



The Pythagorean Theorem: In a right triangle with hypotenuse of length c, and legs of length a and b, then $a^2+b^2=c^2$.



hypotenuse: $(2x+4)+2 = 2x+6$

long leg: $2x+4$

short leg: x

hypotenuse $\xrightarrow[\text{to}]{\text{compare}}$ long leg $\xrightarrow[\text{to}]{\text{compare}}$ short leg

x

Pythagorean Theorem: $a^2+b^2=c^2$
(c= hypotenuse)

$$x^2+(2x+4)^2=(2x+6)^2$$

$$x^2+(2x+4)(2x+4) = (2x+6)(2x+6)$$

$$x^2+4x^2+8x+8x+16 = 4x^2+12x+12x+36$$

$$\underset{-4x^2}{5x^2}+\underset{-24x}{16x}+\underset{-36}{16} = \underset{-4x^2}{4x^2}+\underset{-24x}{24x}+\underset{-36}{36}$$

$$x^2-8x-20=0$$

$$(x-10)(x+2)=0$$

Cont'd next page

$$x^2 - 8x - 20 = 0$$

$$(x - 10)(x+2) = 0$$

$x - 10 = 0$

$x = 10$

$x + 2 = 0$

$x = -2$

Throw out!
negative number does not make sense for a dimensions

Short leg: $x = 10'$
longer leg: $2x + 4 = 2(10) + 4 = 24'$
hypotenuse: $2x + 6 = 2(10) + 6 = 26'$

The legs are 10 ft and 24 ft, and the hypotenuse is 26 ft.

Due Tuesday:

Prereading Assignment:

Read Section 6.1 (pp. 452-456).
Rework Example 3 (p. 454)
Work Matched Problem #3