5.8: Applications of Quadratic Equations (cont'd)

Note Title 4/7/2015

Ex: Find two consecutive integers whose product is 27 more than 5 times the larger integer.

1st integer: x
(smaller)

2nd integer: $x+1$
(larger)

$$\text{Product} = 5(\text{larger integer}) + 27$$
$$x(x+1) = 5(x+1) + 27$$
$$x^2 + x = 5x + 5 + 27$$
$$x^2 + x = 5x + 32$$
$$\quad -5x - 32 \quad -5x \quad -32$$
$$x^2 - 4x - 32 = 0$$

(−) opposite signs
difference of $4x$

$$(x+4)(x-8) = 0$$

$x + 4 = 0$ or $x - 8 = 0$

$x = -4$ | $x = 8$

Scratchwork

x^2: $x \cdot x$

32: $1 \cdot 32$, $2 \cdot 16$, $4 \cdot 8$

$4x$, $8x$

Check:
$(x+4)(x-8)$
$= x^2 - 8x + 4x - 32$
$= x^2 - 4x - 32$ ✓ ok

$x = -4$ | 1st integer: $x = -4$
2nd integer: $x + 1 = -4 + 1 = -3$
(The two integers are -4 and -3.)

$x = 8$ | 1st integer: $x = 8$
2nd integer: $x + 1 = 8 + 1 = 9$
(the two integers are 8 and 9).

$\boxed{\text{The pair of integers is either } -4, -3 \text{ or } 8, 9.}$

Check answers:

$-3, -4$ | consecutive? Yes
Product: $(-3)(-4) = 12$
5 times larger: $5(-3) = -15$
27 more: $-15 + 27 = 12$
✓ same! equal!

$8, 9$ | Consecutive? Yes
Product: $8(9) = 72$
5 times larger: $5(9) = 45$
27 more: $45 + 27 = 72$
✓ equal!

Ex: (5.8 #6)

The product of two consecutive odd integers is "=" 1 less than 4 times their sum. Find the two integers.

1st integer: x

2nd integer: $x+2$

$$\text{Product} = 4(\text{sum}) - 1$$
$$x(x+2) = 4(x+x+2) - 1$$
$$x^2+2x = 4(2x+2)-1$$
$$x^2+2x = 8x+8-1$$
$$x^2+2x = 8x+7$$
$$\quad -8x-7 \qquad -8x \quad -7$$
$$x^2-6x-7=0$$
$$(x-7)(x+1)=0$$

Check:
$(x-7)(x+1)$
$= x^2+1x-7x-7$
$= x^2-6x-7$ ✓ok

$x-7=0$ or $x+1=0$

$x=7$ | $x=-1$

$x=7$) 1st integer: $x=7$
2nd integer: $x+2=7+2=9$
(The two integers are 7 and 9).

$x=-1$) 1st integer: $x=-1$
2nd integer: $x+2=-1+2=1$
(The two integers are -1 and 1).

$\boxed{\text{The pair of integers is } 7, 9 \text{ or } -1, 1.}$

Check:

7, 9) Consecutive odd integers? Yes
Product: $7(9)=63$
Sum: $7+9=16$
4 times sum: $4(16)=64$
1 less: 63

equal! ✓ok

-1, 1) consecutive odd integers? Yes
Product: $(-1)(1)=-1$
Sum: $-1+1=0$
4 times sum: $4(0)=0$
1 less: -1

equal ✓ok

6.1: Simplifying Rational Expressions

Recall: **Rational Number:** can be written as the ratio (fraction, quotient) of 2 integers.

Examples of rational numbers:

$\frac{2}{5}$, $-\frac{7}{2}$, $2\frac{3}{5}$, 5, 0, 0.379, 3.14, $0.\overline{3}$, $2.4\overline{917}$

$2\frac{3}{5} = \frac{13}{5}$, $5 = \frac{5}{1}$, $0 = \frac{0}{1}$, $0.379 = \frac{379}{1000}$, $3.14 = \frac{314}{100}$, $0.\overline{3} = 0.3333\ldots = \frac{1}{3}$, $2.4\overline{917} = 2.4917917917\ldots$

Irrational numbers (not rational)

π, $\sqrt{2}$, $\sqrt{3}$, e

$\pi = \frac{\text{circumference}}{\text{diameter}}$

Rational Expression: can be written as the ratio (quotient, fraction) of 2 polynomials.

Examples of Rational Expressions

$$\frac{x^2+4}{x^3-8}, \quad \frac{x^2+x+24}{x^2-9}, \quad \frac{1}{x^2-2x-8}, \quad \underbrace{x^{-1}+y^{-1}}_{\frac{1}{x}+\frac{1}{y}}$$

Not Rational Expressions

$$\frac{\sqrt{x^2+4}}{x^3-8}, \quad 3x^2-2\sqrt{x}+1, \quad 3^x$$

> To simplify a rational expression, we factor the numerator and denominator, then reduce the fraction by "canceling out" factors that appear in both the numerator and denominator.

Ex: Simplify.

$$\frac{16}{36} = \frac{\cancel{4}\cdot 4}{\cancel{4}\cdot 9} = \frac{4}{4}\cdot\frac{4}{9} = 1\cdot\frac{4}{9} = \boxed{\frac{4}{9}}$$

Ex: Simplify.

$$\frac{\cancel{24}^{3}x^4y^3}{\cancel{32}_{4}x^5y} = \boxed{\frac{3y^2}{4x}}$$

Ex: Simplify.

$$\frac{x^2+8x+15}{x^2+5x+6} = \frac{\cancel{(x+3)}(x+5)}{(x+2)\cancel{(x+3)}}$$

$$= \boxed{\frac{x+5}{x+2}}$$

Restrictions: (denominator cannot be 0)

$x \neq -2$

$x \neq -3$

Scratchwork

$x^2+8x+15$ (+) same

$(x+3)(x+5)$

check

$x^2+5x+3x+15$

$x^2+8x+15$

15

^

1 . 15

3 . 5

x^2+5x+6

$=(x+2)(x+3)$

Chk:

$x^2+3x+2x+6$

$=x^2+5x+6$ ✓

Ex: Simplify.

$$\frac{x-3}{x^2-7x+12} = \frac{\cancel{x-3}}{\cancel{(x-3)}(x-4)} = \boxed{\frac{1}{x-4}}$$

Restrictions: $x \neq 3$

$x \neq 4$

Ex: Simplify.

$$\frac{x-2}{4-x^2} = \frac{x-2}{-x^2+4} = \frac{x-2}{-1(x^2-4)} = \frac{\cancel{x-2}}{-1(x+2)\cancel{(x-2)}}$$

Recommended Method

$$= \frac{1}{-1(x+2)} = \boxed{-\frac{1}{x+2}}$$

Previous example (another way)

$$\frac{x-2}{4-x^2} = \frac{x-2}{(2+x)(2-x)} = \frac{x-2}{(2+x)(-1)(-2+x)}$$

$$= \frac{\cancel{x-2}^{\,1}}{-1(x+2)\cancel{(x-2)}_{1}} = \frac{1}{-1(x+2)} = \boxed{-\frac{1}{x+2}}$$

6.2: Multiplying and Dividing Rational Expressions

Ex: $\frac{2}{3}\cdot\frac{4}{5} = \boxed{\frac{8}{15}}$

Ex: $\frac{16}{15}\cdot\frac{21}{32} = \frac{\cancel{16}^{\,1}}{15_{\,5}}\cdot\frac{\cancel{21}^{\,7}}{\cancel{32}_{\,2}} = \boxed{\frac{7}{10}}$

Ex: Simplify.

$$\frac{x^2+2x}{x^2-4}\cdot\frac{x^2-4x+4}{x^2-x}$$

[Factor numerators & denominators]

$$= \frac{\cancel{x}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x-2)}}\cdot\frac{\cancel{(x-2)}(x-2)}{\cancel{x}(x-1)}$$

$$= \boxed{\frac{x-2}{x-1}}$$

Ex: $\frac{x^2-7x+12}{x^2-25} \div \frac{x^2-3x}{x+5}$

[convert to multiplication (reciprocal)]

$$= \frac{x^2-7x+12}{x^2-25}\cdot\frac{x+5}{x^2-3x}$$

$$= \frac{\cancel{(x-3)}(x-4)}{\cancel{(x+5)}(x-5)}\cdot\frac{\cancel{x+5}}{x\cancel{(x-3)}}$$

$$= \boxed{\frac{x-4}{x(x-5)}}$$