7.1: Solving Linear Systems by Graphing

Note Title 4/21/2015

System of Equations: Group of equations (2 or more)

Solution to a System of Equations: A set of values for the variables that makes *all* of the equations true.

Linear system (of equations): (All variables are to 1st power only) Equations are linear

Examples of linear eqns: $Ax+By=C$, $Ax+By+Cy=D$

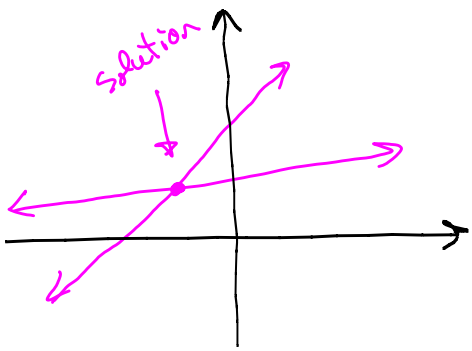
We will be working with 2x2 linear systems:
(2 equations, 2 variables)

Example: $\begin{cases} 2x-5y=4 \\ x+6y=8 \end{cases}$ } graphs of these are lines

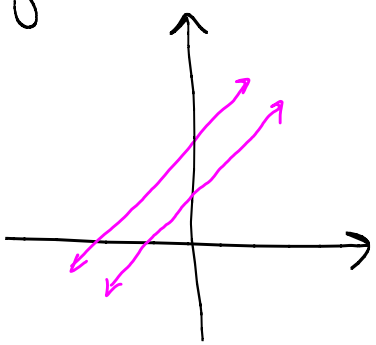
Methods for solving linear systems:

1) Graphing (Section 7.1)
2) Substitution (Section 7.2)
3) Elimination (Section 7.3)
4) Matrix Methods (College Algebra, or Finite Math*)

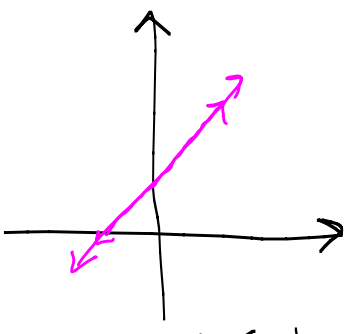
3 possible scenarios for the graph of a linear system with 2 equations in 2 variables



Independent system
One solution
(lines intersect at a single point)



Inconsistent System
No solution
(lines are parallel)



Dependent System
Infinitely many solutions
(both equations represent the same line)

Question on Test #5

For an inconsistent system, a dependent system, and/or an independent system:

1) State the number of solutions
2) Describe the graph in words (lines are parallel, lines are the same, lines intersect at a single point)
3) Illustrate with an example graph.

Verifying whether an ordered pair is a solution to a system:

Example: Is the ordered pair $(9, -2)$ a solution to this system?

$$\begin{cases} 2x + 5y = 8 \\ 3x - 2y = 23 \end{cases}$$

Put $x = 9$, $y = -2$ into $2x + 5y = 8$.

$2(9) + 5(-2) = 8$

$18 - 10 = 8$

$8 = 8$ ✓ True

So $(9, -2)$ is a solution to the 1st equation.

Put $x = 9$, $y = -2$ into $3x - 2y = 23$

$3(9) - 2(-2) = 23$

$27 + 4 = 23$

$31 = 23$ False

So $(9, -2)$ is not a solution to the 2nd eqn.

No, $(9, -2)$ is not a solution to the system.

Ex: Is $(1, -\frac{2}{3})$ a solution to the system?

$$\begin{cases} 2x - 3y = 4 \\ 5x + 6y = 1 \end{cases}$$

$x = 1, y = -\frac{2}{3} \Rightarrow$

$2x - 3y = 4$

$2(1) - 3\left(-\frac{2}{3}\right) = 4$

$2 + \frac{6}{3} = 4$

$2 + 2 = 4$

$4 = 4$ ✓ True

$5x + 6y = 1$

$x = 1, y = -\frac{2}{3} \Rightarrow 5(1) + 6\left(-\frac{2}{3}\right) = 1$

$5 - \frac{12}{3} = 1$

$5 - 4 = 1$

$1 = 1$ ✓ True

Yes, $(1, -\frac{2}{3})$ is a solution to the system.

Example: Solve the system by graphing.

$$\begin{cases} x + 2y = 8 \\ x - 3y = 3 \end{cases}$$

Steps

1) Graph both lines.
2) Estimate the intersection point.
3) Check.

Graph the 1st line $x + 2y = 8$:

Find the x-intercept: set $y = 0$

$y = 0 \Rightarrow x + 2(0) = 8$

$x + 0 = 8$

$x = 8$ ordered pair: $(8, 0)$

Find the y-intercept: set $x = 0$

$x = 0 \Rightarrow 0 + 2y = 8$

$2y = 8$

$y = 4$ ordered pair $(0, 4)$

Graph the 2nd line:

$x - 3y = 3$

Find x-intercept: set $y = 0$:

$y = 0 \Rightarrow x - 3(0) = 3$

$x - 0 = 3$

$x = 3$ ordered pair $(3, 0)$

See next page

Previous example cont'd:

$$x - 3y = 3$$

Find y-intercept: Set $x = 0$:

$x = 0 \Rightarrow \quad 0 - 3y = 3$

$-3y = 3$

$\frac{-3y}{-3} = \frac{3}{-3}$

$y = -1$ ordered pair $(0, -1)$

See graph paper.

Solution appears to be $(6, 1)$.

Check it:

$x + 2y = 8$

$x = 6, y = 1 \Rightarrow \quad 6 + 2(1) = 8$

$6 + 2 = 8$

$8 = 8$ ✓ True

$x - 3y = 3$

$x = 6, y = 1 \Rightarrow \quad 6 - 3(1) = 3$

$6 - 3 = 3$

$3 = 3$ ✓

True

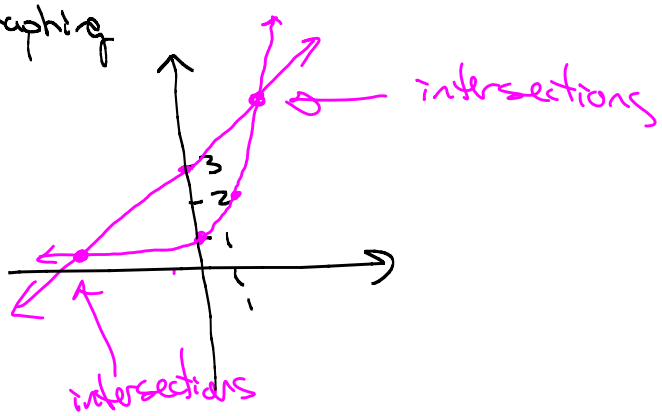
$(6, 1)$ is the solution

Because all we can do from a graph is estimate the solution, the substition and elimination methods are more practical.

Note: Some (non linear) systems can only be solved by graphing

$y = 2^x$

$y = x + 3$

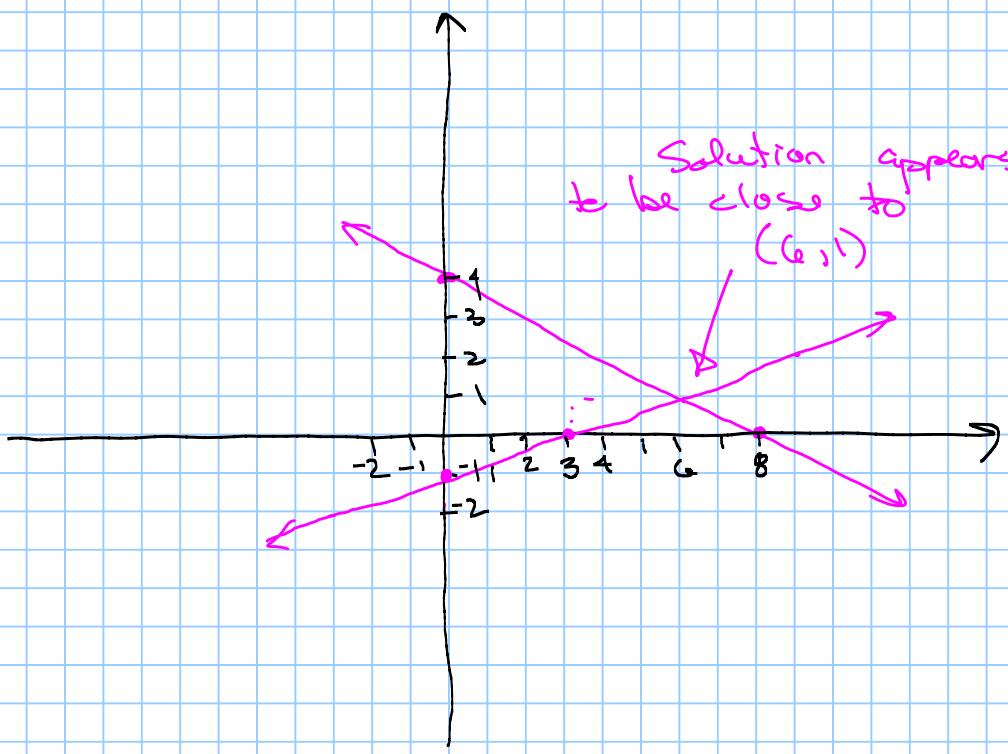


Cannot find the solution algebraically.

Graph from Section 7.1 Example:

1st line: $x+2y=8$
$(8,0)$
$(0,4)$

2nd line: $x-3y=3$
$(3,0)$
$(0,-1)$



7.2: The Substitution Method
(for solving linear systems)

Solving by substitution: step-by-step

1) Solve one equation (your choice) for one variable (your choice)

2) Substitute this result into the other equation.

3) Solve for the remaining variable.

4) Substitute this value into the either equation and solve.

5) Check it!

Example: Solve the system by substitution.

$$\begin{cases} x - y = 9 \\ 2x - 11y = -18 \end{cases}$$

1) Solve $x - y = 9$ for x.

$$x = 9 + y$$

2) Substitute $x = 9 + y$ into the other equation.

$$2x - 11y = -18$$

$$2(9+y) - 11y = -18$$

3) Solve for y:

$$18 + 2y - 11y = -18$$

$$18 - 9y = -18$$

$$-18 \quad\quad -18$$

$$-9y = -36$$

$$\frac{-9y}{-9} = \frac{-36}{-9}$$

$$y = 4$$

4) Put $y = 4$ into $x - y = 9$.

$$x - 4 = 9$$

$$x = 13$$

Solution is $(13, 4)$.

(Check on next page)

5) Check it!

$x - y = 9$

$x=13, y=4 \Rightarrow 13-4=9$

$9=9$ ✓ ok

$2x-11y=-18$

$x=13, y=4 \Rightarrow 2(13)-11(4)=-18$

$26-44=-18$

$-18=-18$ ✓ True

$44-26=18$

Ex: Solve by substitution.

$$\begin{cases} -4x+3y=19 \\ -6x-5y=0 \end{cases}$$

1) solve $-6x-5y=0$ for y.

$-5y=6x$

$\frac{-5y}{-5}=\frac{6x}{-5}$

$y=-\frac{6}{5}x$

2) substitute $y=-\frac{6}{5}x$ into the other eqn:

$-4x+3y=19$

$-4x+3\left(-\frac{6}{5}x\right)=19$

3) Solve for x:

$-4x-\frac{18}{5}x=19$

$-\frac{4x}{1}\left(\frac{5}{5}\right)-\frac{18}{5}x=19$

$-\frac{20}{5}x-\frac{18}{5}x=19$

$-\frac{38}{5}x=19$

$-\frac{5}{38}\left(-\frac{38}{5}x\right)=(19)\left(-\frac{5}{38}\right)$

$x=-\frac{5}{2}$

19 × 2 = 38

4) Put $x=-\frac{5}{2}$ into either of the original eqns, or put into $y=-\frac{6}{5}x$

$y=-\frac{6}{5}\left(-\frac{5}{2}\right)=+\frac{30}{10}=3$

Solution is: $\left(-\frac{5}{2}, 3\right)$.

Check it! (see next page)

Check of previous problem

$$-4x+3y=19$$

$x=-\frac{5}{2}$, $y=3 \Rightarrow -4\left(-\frac{5}{2}\right)+3(3)=19$

$$\frac{20}{2}+9=19$$

$$10+9=19$$

$$19=19$$

$$-6x-5y=0$$

$x=-\frac{5}{2}$, $y=3 \Rightarrow -6\left(-\frac{5}{2}\right)-5(3)=0$

$$15-15=0$$

$$0=0 \checkmark$$