7.3: The Elimination Method (for solving systems)

Note Title 4/23/2015

The Elimination methods (steps)

1) Multiply both sides of one or both equations by a strategic number.

2) Add the equations to eliminate a variable.

3) Solve the equation.

4) Eliminate the other variable by:

   a) substituting the known value into either of the original equations and then solving

   OR b) doing elimination again on the 2nd variable

5) Check your solution.

Example: Solve the system by elimination.

$$\begin{cases} 5x - 12y = 1 \\ -x + 6y = -2 \end{cases}$$

$5x - 12y = 1 \longrightarrow$ $5x - 12y = 1$

$-x + 6y = -2 \xrightarrow{(2)}$ $-2x + 12y = -4$

(multiply both sides of 2nd equation by 2)

Add: $3x + 0y = -3$

$3x + 0 = -3$

$3x = -3$

$\frac{3x}{3} = \frac{-3}{3}$

$x = -1$

Put $x = -1$ into $-x + 6y = -2$

$-(-1) + 6y = -2$

$1 + 6y = -2$

$6y = -3$

$\frac{6y}{6} = \frac{-3}{6}$

$y = -\frac{1}{2}$

Solution is $(-1, -\frac{1}{2})$

Check on next page

<u>Check:</u> $5x - 12y = 1$

$x = -1, y = -\frac{1}{2} \Rightarrow 5(-1) - 12\left(-\frac{1}{2}\right) = 1$

$-5 + \frac{12}{2} = 1$

$-5 + 6 = 1$

$1 = 1$ ✓

$-x + 6y = -2$

$x = -1, y = -\frac{1}{2} \Rightarrow -(-1) + 6\left(-\frac{1}{2}\right) = -2$

$1 - \frac{6}{2} = -2$

$1 - 3 = -2$

$-2 = -2$ ✓

Example: Solve by elimination.

$$11x - 5y = 2$$
$$-y + 3x = 1$$

Rearrange: (so like terms line up)

$$\begin{aligned} 11x - 5y &= 2 &\longrightarrow\quad 11x - 5y &= 2 \\ 3x - y &= 1 &\xrightarrow{(-5)}\quad -15x + 5y &= -5 \end{aligned}$$

Add $-4x + 0 = -3$

$$-4x = -3$$
$$\frac{-4x}{-4} = \frac{-3}{-4}$$
$$x = \frac{3}{4}$$

Put $x = \frac{3}{4}$ into either equation, or do elimination again to find y.

$$\begin{aligned} 11x - 5y &= 2 &\xrightarrow{(-3)}\quad -33x + 15y &= -6 \\ 3x - y &= 1 &\xrightarrow{(11)}\quad 33x - 11y &= 11 \end{aligned}$$

Add: $4y = 5$

$$\frac{4y}{4} = \frac{5}{4}$$
$$y = \frac{5}{4}$$

$\boxed{\text{The solution is } \left(\frac{3}{4}, \frac{5}{4}\right).}$

Check:

$x = \frac{3}{4}, y = \frac{5}{4} \Rightarrow$

$$11x - 5y = 2$$
$$11\left(\frac{3}{4}\right) - 5\left(\frac{5}{4}\right) = 2$$
$$\frac{33}{4} - \frac{25}{4} = 2$$
$$\frac{8}{4} = 2$$
$$2 = 2 \checkmark$$

$\left.\begin{array}{l} x = \frac{3}{4} \\ y = \frac{5}{4} \end{array}\right\} \Rightarrow$

$$3x - y = 1$$
$$3\left(\frac{3}{4}\right) - \frac{5}{4} = 1$$
$$\frac{9}{4} - \frac{5}{4} = 1$$
$$\frac{4}{4} = 1$$
$$1 = 1 \checkmark$$

Ex: $7x - 8y = -7$
$5x + 9y = 6$

$$\begin{aligned}
7x - 8y &= -7 \xrightarrow{(9)} & 63x - 72y &= -63 \\
5x + 9y &= 6 \xrightarrow{(8)} & 40x + 72y &= 48 \\
\hline
& & 103x &= -15
\end{aligned}$$

$$\frac{103x}{103} = \frac{-15}{103}$$

$$x = -\frac{15}{103}$$

$$\begin{aligned}
7x - 8y &= -7 \xrightarrow{(-5)} & -35x + 40y &= 35 \\
5x + 9y &= 6 \xrightarrow{(7)} & 35x + 63y &= 42 \\
\hline
\text{Add:} & & 103y &= 77
\end{aligned}$$

$$\frac{103y}{103} = \frac{77}{103}$$

$$y = \frac{77}{103}$$

The solution is $\boxed{\left(-\frac{15}{103}, \frac{77}{103}\right).}$

Example: Solve.

$-x + 6y = 2$
$3x - 18y = 5$

$$\begin{aligned}
-x + 6y &= 2 \xrightarrow{(3)} & -3x + 18y &= 6 \\
3x - 18y &= 5 \longrightarrow & 3x - 18y &= 5 \\
\hline
\text{Add:} & & 0x + 0y &= 11 \\
& & 0 &= 11
\end{aligned}$$

False statement

$\boxed{\text{No Solution}}$

(system is <u>inconsistent</u>, lines are parallel)

If we did the previous problem with substitution method, we get:

$$\begin{cases} -x+6y=2 \\ 3x-18y=5 \end{cases}$$

$-x+6y=2$

solve for x: $-x = 2-6y$

$x = -2+6y$

Put $x=-2+6y$ into $3x-18y=5$

$3(-2+6y)-18y=5$

$-6+18y-18y=5$

$-6=5$ False

$\boxed{\text{No solution. System is inconsistent.}}$

Example: Solve.

$$\begin{cases} 2x-10y=6 \\ -5x+25y=-15 \end{cases}$$

$2x-10y=6 \xrightarrow{(5)} 10x-50y=30$

$-5x+25y=-15 \xrightarrow{(2)} -10x+50y=-30$

Add: $0x+0y=0$

$0=0$ True.

$\boxed{\text{Dependent System (Infinitely Many Solutions)}}$

(equations are the same line)

Summary: How to recognize inconsistent systems and dependent systems:

When solving a system by elimination or substitution, and both variables disappear:

1) If we get a false statement (such as $0 = 11$ or $-6 = 5$), the system is inconsistent and has no solutions. (lines are parallel)

2) If we get a true statement (such as $0 = 0$ or $5 = 5$), the system is dependent and has infinitely many solutions. (lines are the same)

Hint: If the problem has fractions, get rid of them by multiplying by the LCD.

Ex.:

$$\frac{x}{12} - \frac{y}{15} = \frac{-3}{20}$$

$$-\frac{x}{6} + \frac{y}{8} = \frac{5}{24}$$

1st eqn: LCD: 60

$$\frac{x}{12} - \frac{y}{15} = \frac{-3}{20} \xrightarrow{(60)} \frac{\cancel{60}^{5}}{1}\left(\frac{x}{\cancel{12}_{1}}\right) - \frac{y}{\cancel{15}_{1}}\frac{(\cancel{60}^{4})}{1} = -\frac{3}{\cancel{20}_{1}}\frac{(\cancel{60}^{3})}{1}$$

$$5x - 4y = -9$$

2nd eqn: 24

$$-\frac{x}{6} + \frac{y}{8} = \frac{5}{24} \xrightarrow{(24)} -\frac{x}{\cancel{6}_{1}}\frac{(\cancel{24}^{4})}{1} + \frac{y}{\cancel{8}_{1}}\frac{(\cancel{24}^{3})}{1} = \frac{5}{\cancel{24}_{1}}\frac{(\cancel{24}^{1})}{1}$$

$$-4x + 3y = 5$$

New system $\begin{cases} 5x - 4y = -9 \\ -4x + 3y = 5 \end{cases}$

7.4: Applications of Linear Systems

Ex: The difference of two positive numbers is 8. The larger number is 7 less than twice the smaller number. Find the numbers.

larger number: x
smaller number: y

2 variables, so we need 2 equations.

1st sentence: $x - y = 8$

2nd sentence: larger # = 2(smaller #) − 7

$$x = 2y - 7$$

System $\begin{cases} x - y = 8 \\ x = 2y - 7 \end{cases}$

Substitution: Substitute $x = 2y - 7$ into $x - y = 8$

$$(2y-7) - y = 8$$
$$2y - 7 - y = 8$$
$$y - 7 = 8$$
$$y = 15$$

Put $y = 15$ into $x - y = 8$

$$x - 15 = 8$$
$$\quad +15 \quad +15$$
$$x = 23$$

$\boxed{\text{The numbers are 15 and 23.}}$

Ex: Mike has \$1.55 in dimes and nickels. He has 7 more nickels than dimes. How many of each does he have?

number of nickels: n
number of dimes: d

2 variables, so need 2 equations.

1st sentence: $\$0.05n + \$0.10d = \$1.55$

multiply by 100 to clear the decimals: $5n + 10d = 155$

2nd sentence: # nickels = # dimes + 7
$n = d + 7$

System: $\begin{cases} n = d+7 \\ 5n + 10d = 155 \end{cases}$

Substitute $n = d+7$ into $5n + 10d = 155$

$$5(d+7) + 10d = 155$$
$$5d + 35 + 10d = 155$$
$$15d + 35 = 155$$
$$-35 \qquad -35$$
$$15d = 120$$
$$\frac{15d}{15} = \frac{120}{15}$$
$$d = 8$$

$$\begin{array}{r} {}^{3}15 \\ \times\ 7 \\ \hline 105 \end{array} \qquad \begin{array}{r} {}^{4}15 \\ \times\ 8 \\ \hline 120 \end{array}$$

$n = d+7$
$d = 8 \Rightarrow n = 8 + 7 = 15$

$\boxed{\text{He has 15 nickels and 8 dimes.}}$

$$\begin{array}{r} {}^{2}15 \\ \times\ 5 \\ \hline 75 \end{array}$$

Check: 8 dimes: \$0.80
15 nickels: 0.75
$\overline{\ 1.55\ }$ ✓