Homework Qs

Note Title 1/21/2015

1.3 #41

$\lim_{x \to 0} \frac{x^2+3x}{x}$ Question does not make sense as written.

Do this one instead.

Write a simpler fcn that agrees with the given fcn at all but one point.

$f(x) = \frac{x^2+3x}{x}$ Note: f is undefined at 0.

Domain of f: $(-\infty, 0) \cup (0, \infty)$

$f(x) = \frac{x(x+3)}{x}$

Important: $\frac{x(x+3)}{x} \neq x+3$ for all x

So $f(x) \neq x+3$

Similar function: $\boxed{g(x) = x+3}$ Note: Domain of g: $(-\infty, \infty)$

f and g cannot be the same function, because they have different domains.

$\lim_{x \to 0} g(x) = \lim_{x \to 0} (x+3) = 0+3 = \boxed{3}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x(x+3)}{x} = \lim_{x \to 0} (x+3) = 0+3 = \boxed{3}$

These functions differ only at 0.

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1.3 #41 cont'd

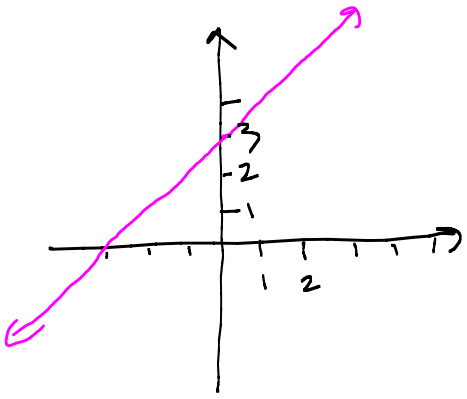
Let's graph them:

$g(x) = x + 3$

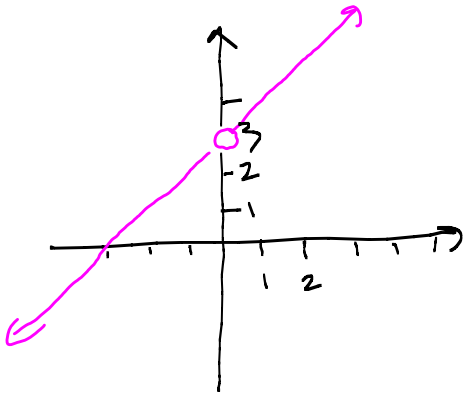
$y = 1x + 3$

y-intercept: 3

slope: $1 = +\frac{1}{1}$



$f(x) = \frac{x(x+3)}{x}$



1.3 #61

$$\lim_{x \to 0} \frac{\sin^2 x}{x}$$

$$= \lim_{x \to 0} \left[\left(\frac{\sin x}{x} \right) (\sin x) \right]$$

$$= \left(\lim_{x \to 0} \frac{\sin x}{x} \right) \left(\lim_{x \to 0} \sin x \right)$$

$$= (1)(\sin 0) \quad \leftarrow \text{Theorem 1.6 p 61}$$

$$= (1)(0)$$

$$= \boxed{0}$$

Use this fact:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Note that this implies

$$\lim_{x \to 0} \frac{x}{\sin x} = \frac{1}{1} = 1$$

1.3 #87) $f(x) = \frac{1}{x+3}$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Note:
Δx = "delta x"
= "change in x"

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x+3} - \frac{1}{x+3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[\left(\frac{1}{\Delta x} \right) \cdot \left(\frac{1}{x+\Delta x+3} - \frac{1}{x+3} \right) \right]$$

$$= \lim_{\Delta x \to 0} \left[\frac{1}{\Delta x} \left(\frac{1}{x+\Delta x+3} \left(\frac{x+3}{x+3} \right) - \frac{1}{x+3} \left(\frac{x+\Delta x+3}{x+\Delta x+3} \right) \right) \right]$$