13.2: Limits and Continuity

We will skip the 3-dimensional version of the ε-δ definition of a limit.

Main principle to remember for determining whether $\lim_{(x,y)\to(a,b)} f(x,y)$ exists:

If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ is to be true, then the values of $f(x,y)$ must approach L as (x,y) approaches (a,b) regardless of path. (No matter what path (x,y) follows when approaching (a,b), the function values still approach L.)

In other words, if $f(x,y)\to L_1$ as $(x,y)\to(a,b)$ along the curve C_1, but $f(x,y)\to L_2$ as $(x,y)\to(a,b)$ along the curve C_2, and $L_1 \neq L_2$, then $\lim_{(x,y)\to(a,b)} f(x,y)$ does not exist.

Example 1: Find $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$, if it exists.

Example 2: Find $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$, if it exists.

Example 3: Find $\lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^2+y^2}$, if it exists.

Example 4: Find $\lim_{(x,y)\to(2,1)} \dfrac{xy}{x^2+y^2}$, if it exists.

Continuity:

Definition: A function *f* of two variables is continuous at a point (a,b) in an open region *R* if

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b) \ .$$

The function is continuous on the open region *R* if it is continuous at every point in *R*.

In general, if we combine continuous functions using addition, multiplication, composition, or division, the combined functions are continuous at the places they are defined. (When dividing functions, we may introduce discontinuities due to 0 denominators.)

Example 5: Discuss the continuity of $f(x,y) = \dfrac{xy}{x^2+y^2}$.

Example 6: Discuss the continuity of $f(x,y,z) = \dfrac{\sqrt{y}}{x^2-y^2+z^2}$.