12.3: Velocity and Acceleration

Suppose an object's position is given by the vector-valued function $\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}$. Then the first and second derivatives represent the velocity and acceleration. Note that the velocity and acceleration are vectors that have both magnitude and direction.

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Definition: Suppose *x*, *y*, and *z* are twice-differentiable functions of *t*, and $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$. Then the velocity vector, acceleration vector, and speed at time *t* are:

Velocity = $\mathbf{v}(t) = \mathbf{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$.

Acceleration = $a(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = \langle x''(t), y''(t), z''(t) \rangle$.

Speed = $\|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}$.

(This applies to vectors in $\mathbb{R}^2$ also.)

Note: The velocity vector is also known as the tangent vector. When its initial point is placed at the point on the curve, it is tangent to the curve.

Example 1: Suppose an object's position is given by the vector $\mathbf{r}(t) = \langle t^2, t^3 \rangle$. Sketch the path of the object, and the velocity and acceleration vectors at the point $(1,1)$.

Example 2: Suppose an object's position is given by the vector $\mathbf{r}(t) = \langle t^2, t, t^{3/2} \rangle$. Find the velocity vector, the acceleration vector, and the speed in terms of *t*.

Projectile motion:

Theorem:
If a projectile is launched from an initial height h with an initial speed v_0 and angle of elevation θ, and if air resistance is neglected, then the projectile's path is described by the vector function

$$\mathbf{r}(t) = \left\langle (v_0 \cos\theta)t, h + (v_0 \sin\theta)t - \frac{1}{2}gt^2 \right\rangle,$$

where g is the acceleration of gravity.

Example 3: Suppose a projectile is fired at a height of 3000 feet above the ground with an initial velocity of 900 feet/second at an angle of 45 degrees above the horizontal. Determine the maximum height and range of the projectile.

Example 4: Suppose a projectile is fired from ground level at an angle of 12° above the horizontal. Determine the minimum initial velocity necessary for the projectile to have a range of 200 feet.

Example 5: A baseball player at second base throws a ball 90 feet to the player at first base. The ball is released at 3 feet above the ground with an initial velocity of 70 miles per hour, at an angle of 15° above the horizontal. At what height does the first baseman catch the ball?