14.5: Surface Area

We can use a double integral to find the upper surface area of a solid defined by $z = f(x, y)$ over a region *R*.

> Definition: If *f* and its first partial derivatives are continuous on the closed region *R* in the *xy*-plane, then the area of the surface S given by $z = f(x, y)$ over R is defined as
>
> $$\text{Surface Area} = \iint_R dS$$
> $$= \iint_R \sqrt{1 + [f_x(x,y)]^2 + [f_y(x,y)]^2}\, dA.$$

Note:

Length on *x*-axis: $\int_a^b x\,dx$

Arc length in *xy*-plane: $\int_a^b \sqrt{1+[f'(x)]^2}\,dx$

Area in *xy*-plane: $\iint_R dA$

Surface area in $\mathbb{R}^3$: $\iint_R \sqrt{1 + [f_x(x,y)]^2 + [f_y(x,y)]^2}\, dA$

Example 1: Find the area of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices $(0,0)$, $(0,1)$, and $(2,1)$.

Example 2: Find the area of the surface $f(x, y) = 12 + 2x - 3y$ that lies above the region R bounded by the graph of $x^2 + y^2 \leq 9$.

Example 3: Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$.

Example 4: Find the surface area of the portion of the paraboloid $z = 16 - x^2 - y^2$ that lies in the first octant.

<u>Example 5:</u> Find the surface area of the portion of the hemisphere $z = \sqrt{25 - x^2 - y^2}$ that lies outside the cylinder $x^2 + y^2 = 4$.