2.5: Applications (continued)

Note Title 9/9/2015

2.5 #28 One side of a triangle is 6 m more than twice the shortest side. The third side is 9 m more than the shortest side. The perimeter is 75 m. Find the side lengths.

"one side": $2x+6$

shortest side: x

3rd side: $x+9$

one side —compared to→ shortest side (x)

3rd side —compare to→ shortest side (x)

$$2x+6+x+x+9=75$$

$$4x+15=75$$

$$-15 \quad -15$$

$$4x=60$$

$$\frac{4x}{4}=\frac{60}{4}$$

$$x=15$$

shortest side: $x=15$

one side: $2x+6$

$x=15 \Rightarrow 2(15)+6=30+6=36$

3rd side: $x+9$

$x=15 \Rightarrow 15+9=24$

The sides are 15 m, 36m, and 24m.

Check: Twice shortest side: $2(15)=30$ } 1st sentence checks

6 more: 36 ✓

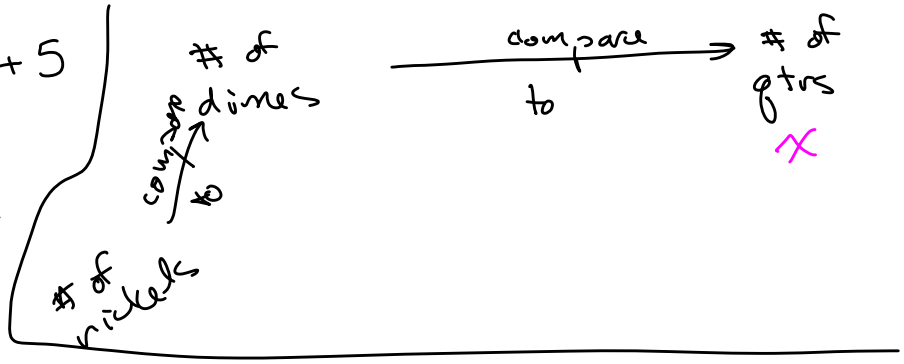
9 more than shortest side: $15+9=24$ ✓ 2nd sentence is OK

Perimeter = $15m+36m+24m=75\ m$ ✓

$$\begin{array}{r} 1 \\ 15 \\ 36 \\ 24 \\ \hline 75 \end{array}$$ ✓ 3rd sentence

Ex: Mary has \$3.00 in nickels, dimes, and quarters. She has twice as many dimes as quarters, and five more nickels than dimes. How many coins of each type does she have?

number of nickels: $2x+5$
# of dimes: $2x$
# of quarters: x



$\$0.05(2x+5) + \$0.10(2x) + \$0.25(x) = \$3.00$

- $\$0.05(2x+5)$: value of all nickels
- $\$0.10(2x)$: value of all dims
- $\$0.25(x)$: value of all qtrs
- $\$3.00$: total value

multiply both sides by 100 to clear the decimals:

$$5(2x+5) + 10(2x) + 25x = 300$$
$$10x + 25 + 20x + 25x = 300$$
$$55x + 25 = 300$$
$$55x = 275$$
$$\frac{55x}{55} = \frac{275}{55}$$
$$x = 5$$

$$\begin{array}{r} 55 \\ \times\ 5 \\ \hline 275 \end{array}$$

# of qtrs: $x=5$

# of nickels: $2x+5$
$x=5 \Rightarrow 2(5)+5 = 10+5 = 15$

# of dimes: $2x$
$x=5 \Rightarrow 2(5) = 10$

She has 5 quarters, 10 dimes, and 15 nickels.

Check on next page.

Check: Total value:

5 qtrs ⇒ $1.25

10 dimes ⇒ $1.00

15 nickels ⇒ $0.75

Add: $3.00 ✓ 1st sentence checks

$$\begin{array}{r} 2 \\ 15 \\ \times\ 5 \\ \hline 75 \end{array}$$

Twice as many dimes as qtrs? Yes ✓

5 more nickels than dimes? Yes

Ex: Joe has $2.25 in nickels, dimes, and quarters. The number of nickels is four times the number of dimes, and there are two more dimes than quarters. How many coins of each type does he have?

# of nickels: $4(x+2) = 4x+8$

# of dimes: $x+2$

# of quarters: x

# of nickels → compare to → # of dimes → compare to → # of qtrs

x

$$\underbrace{\$0.05(4x+8)}_{\text{total value of nickels}} + \underbrace{\$0.10(x+2)}_{\text{total value of dimes}} + \underbrace{\$0.25(x)}_{\text{total value of quarters}} = \$2.25$$

Multiply by 100:

$$5(4x+8) + 10(x+2) + 25x = 225$$

$$20x + 40 + 10x + 20 + 25x = 225$$

$$55x + 60 = 225$$

$$55x = 165$$

$$\frac{55x}{55} = \frac{165}{55}$$

$$x = 3$$

$$\begin{array}{r} 225 \\ -\ 60 \\ \hline 165 \end{array}$$

$$\begin{array}{r} 55 \\ \times\ 3 \\ \hline 165 \end{array} \checkmark$$

# of qtrs: $x = 3$

# of dimes: $x+2$

$x = 3 \Rightarrow 3+2 = 5$

# of nickels: $4x+8$

$x = 3 \Rightarrow 4(3)+8 = 12+8 = 20$

Joe has 3 quarters, 5 dimes, and 20 nickels.

check: 3 qtrs: $0.75
5 dmes: $0.50
20 nickels: $1.00
$2.25

nickels 4 times # dimes? ✓ yes
2 more dimes than qtrs? ✓ yes ✓

2.6: More applications

Consecutive integer problems

Ex: Find 4 consecutive integers that have a sum of 74.

1st integer: x
2nd integer: $x+1$
3rd integer: $x+2$
4th integer: $x+3$

$$x + (x+1) + (x+2) + (x+3) = 74$$
$$x + x+1 + x+2 + x+3 = 74$$
$$4x + 6 = 74$$
$$-6 \quad -6$$
$$4x = 68$$
$$\frac{4x}{4} = \frac{68}{4}$$
$$x = 17$$

1st integer: $x = 17$
2nd integer: $x+1 = 17+1 = 18$
3rd integer: $x+2 = 17+2 = 19$
4th integer: $x+3 = 17+3 = 20$.

The integers are 17, 18, 19, and 20.

Check: Consecutive? Yes

Sum

17
18
19
20

74

Ex: Find three consecutive even integers whose sum is 120.

1st integer: x
2nd integer: $x+2$
3rd integer: $x+4$

$$x + (x+2) + (x+4) = 120$$
$$3x + 6 = 120$$
$$3x = 114$$
$$\frac{3x}{3} = \frac{114}{3}$$
$$x = 38$$

$$3\overline{)114}\quad 38$$
9
24
24
0

1st integer: $x = 38$
2nd integer: $x+2 = 38+2 = 40$
3rd integer: $x+4 = 38+4 = 42$

The integers are 38, 40, and 42.

consecutive even? ✓
sum ..

$$38 + 40 + 42 = 120 \checkmark$$

Note:

Consecutive odd integers:

1st integer: x
2nd integer: $x+2$
3rd: $x+4$
4th: $x+6$

Consecutive odd integers are always 2 units apart

2.7: Linear Inequalities

Inequality symbols

$<$ means "is less than"

$>$ means "is greater than"

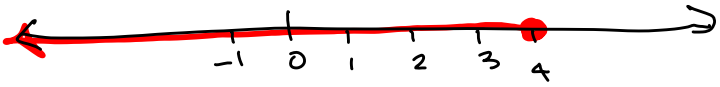
$\leq$ or $\leq$ means "is less than or equal to"

$\geq$ or $\geq$ means "is greater than or equal to"

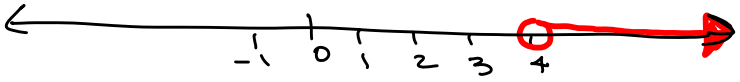
We use number lines to represent inequalities that have variables in them.

$x < 4$

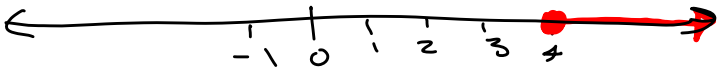
$x \leq 4$



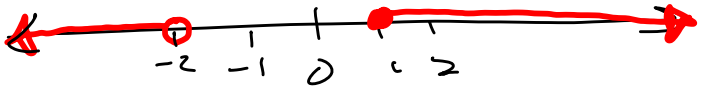
same as $4 < x$ → $x > 4$



same as $4 \leq x$ → $x \geq 4$



$x < -2$ or $x \geq 1$



$-2 < x \leq 1$

this means $-2 < x$ and $x \leq 1$

$x > -2$ and $x \leq 1$