

Section 2.7: Inequalities (cont'd)

Note Title

9/16/2015

Example: Solve.

$$1 - 3x \leq x + 11$$

$$1 - 4x \leq 11$$

$$-4x \leq 10$$

$$\frac{-4x}{-4} \geq \frac{10}{-4} \quad [\text{Reverse the inequality sign}]$$

$$x \geq -\frac{5}{2}$$

$$x \geq -2\frac{1}{2}$$



Ex:

$$6(3-x) > 2-(3x+1)$$

$$18-6x > 2-3x-1$$

$$18-6x > 1-3x$$

$$18-6x+3x > 1$$

$$18-3x > 1$$

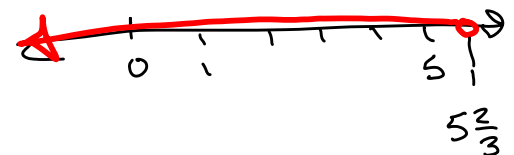
$$-3x > 1-18$$

$$-3x > -17$$

$$\frac{-3x}{-3} < \frac{-17}{-3} \quad [\text{Reverse the inequality sign}]$$

$$x < \frac{17}{3}$$

$$x < 5\frac{2}{3}$$



Ex:

Solve.

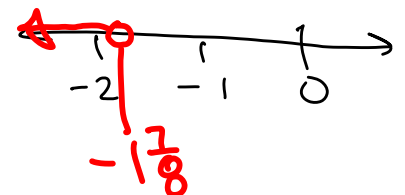
$$-\frac{2}{3}x > \frac{5}{4}$$

$$\left(-\frac{3}{2}\right)\left(-\frac{2}{3}x\right) < \frac{5}{4}\left(-\frac{3}{2}\right)$$

$$x < -\frac{15}{8}$$

$$x < -1\frac{7}{8}$$

Reverse the inequality!



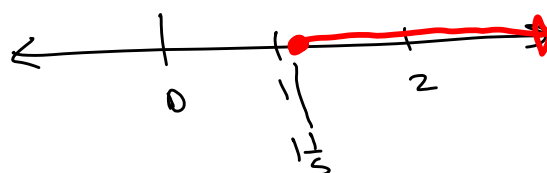
$$6 - x \leq 4x$$

$$6 \leq 5x$$

$$\frac{6}{5} \leq x$$

$$x \geq \frac{6}{5}$$

$$x \geq 1\frac{1}{5}$$



Ex.:

$$\frac{3}{4}x - \frac{1}{3} < 4 + \frac{1}{2}x + \frac{2}{3}x$$

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$$\frac{3}{4}x - \frac{1}{2}x - \frac{2}{3}x < \underline{4} + \frac{1}{3}$$

$$\frac{9}{12}x - \frac{6}{12}x - \frac{8}{12}x < \frac{12}{3} + \frac{1}{3}$$

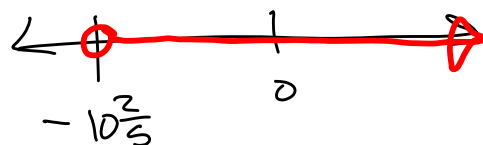
$$-\frac{5}{12}x < \frac{13}{3}$$

Reverse
the
inequality

$$\left(-\frac{12}{5}\right)\left(-\frac{5}{12}x\right) > \frac{13}{3}\left(-\frac{12}{5}\right)$$

$$x > -\frac{52}{5}$$

$$x > -10\frac{2}{5}$$



Combined Inequalities

(Two inequalities combined into 1 statement)

Example:

$$x + 3 < 2$$

or

$$2x + 1 > 6$$

$$x < 2 - 3$$

$$2x > 5$$

$$x < -1$$

$$\frac{2x}{2} > \frac{5}{2}$$

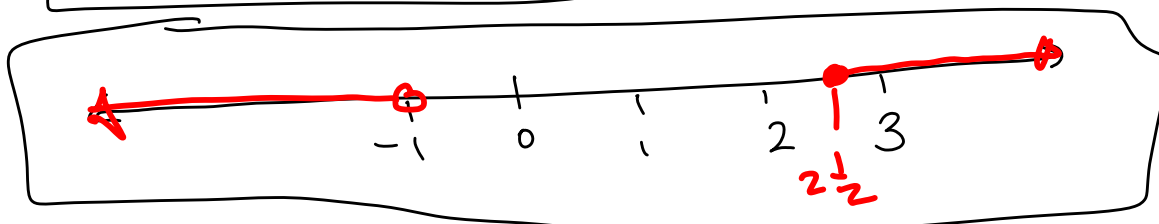
$$x > \frac{5}{2}$$

Answer:

$$x < -1 \text{ or } x > \frac{5}{2}$$

$$x < -1 \text{ or } x > 2\frac{1}{2}$$

either one is okay



$$x < -1$$

$$x > 2\frac{1}{2}$$

Ex:

Solve.

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$$-8 \leq 2x+1 < 17$$

This means:

$$-8 \leq 2x+1 \quad \text{and} \quad 2x+1 < 17$$

$$-9 \leq 2x$$

$$2x < 16$$

$$\frac{-9}{2} \leq \frac{2x}{2}$$

$$\frac{2x}{2} < \frac{16}{2}$$

$$-\frac{9}{2} \leq x$$

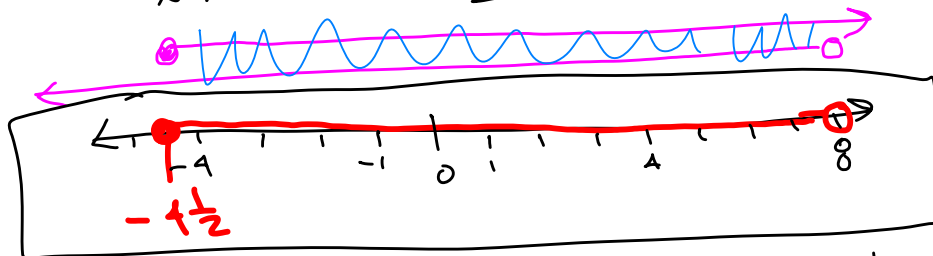
$$x < 8$$

$$x \geq -\frac{9}{2}$$

$$x \geq -4\frac{1}{2}$$

$$\text{and} \quad x < 8$$

want the
x's that
satisfy
both.



Put the inequalities back together into a combined inequality:

$$x \geq -4\frac{1}{2} \quad \text{and} \quad x < 8$$

$$-4\frac{1}{2} \leq x \quad \text{and} \quad x < 8$$

$$\boxed{-4\frac{1}{2} \leq x < 8}$$

Final answer

A more compact way to do this problem:

See next page

Previous
example
cont'd:

$$-9 \leq 2x+1 < 17$$

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[Subtract 1 from all 3 sides]

$$-9 \leq 2x < 16$$

$$\frac{-9}{2} \leq \frac{2x}{2} < \frac{16}{2} \quad [\text{Divide all 3 sides by 2}]$$

$$-\frac{9}{2} \leq x < 8$$

$$\boxed{-4\frac{1}{2} \leq x < 8}$$



Ex.:

Solve $-2 < -6x-2 \leq 5$

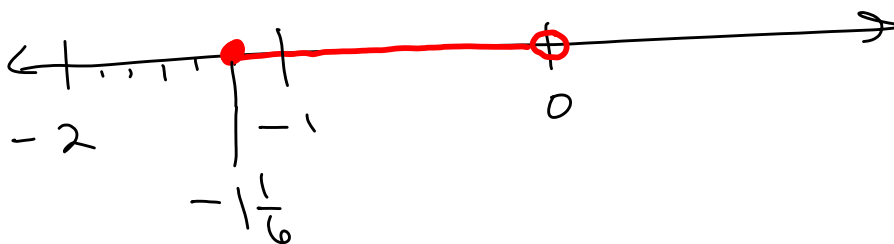
$$0 < -6x \leq 7$$

$$\frac{0}{-6} > \frac{-6x}{-6} \geq \frac{7}{-6}$$

Reverse both the inequality signs

$$0 > x \geq -\frac{1}{6}$$

$$\boxed{-\frac{1}{6} \leq x < 0}$$



Ex.:

Solve.

⑤

$$-10 \leq 2x+3 < -14$$

-3 -3 -3

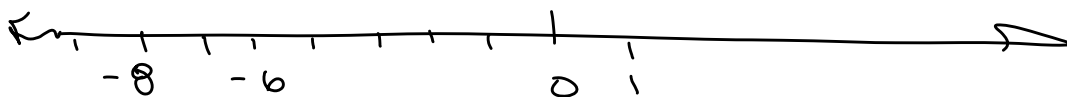
$$-13 \leq 2x < -17$$

$$-\frac{13}{2} \leq \frac{2x}{2} < -\frac{17}{2}$$

$$-6\frac{1}{2} \leq x < -8\frac{1}{2}$$

$$-6\frac{1}{2} \leq x \text{ and } x < -8\frac{1}{2}$$

$$x \geq -6\frac{1}{2}$$



x cannot satisfy both of these!

No Solution

Ex.:

$$-\frac{2}{5} \leq \frac{1}{4} \left(7x + \frac{1}{3} \right) < \frac{2}{3}$$

$$-\frac{2}{5} \leq \frac{1}{4} \left(7x \right) + \frac{1}{4} \left(\frac{1}{3} \right) < \frac{2}{3}$$

$$-\frac{2}{5} \leq \frac{7}{4}x + \frac{1}{12} < \frac{2}{3}$$

$$-\frac{2 \cdot 12}{5 \cdot 12} - \frac{1 \cdot 5}{12 \cdot 5} \leq \frac{7}{4}x < \frac{2}{3} - \frac{1}{12}$$

$$-\frac{24}{60} - \frac{5}{60} \leq \frac{7}{4}x < \frac{8}{12} - \frac{1}{12}$$

$$-\frac{29}{60} \leq \frac{7}{4}x < \frac{7}{12}$$

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page

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$$\begin{array}{r} 3 \\ 15 \\ \hline 106 \end{array}$$

$$-\frac{29}{60} \leq \frac{1}{x} < \frac{1}{12}$$

$$-\frac{29}{60} \left(\frac{4}{7} \right) \leq \frac{1}{x} \times \left(\frac{4}{7} \right) < \frac{1}{12} \left(\frac{4}{7} \right)$$

15 3

$$-\frac{29}{105} \leq x < \frac{1}{3}$$